ie. $x^2 (1397^2+4835^2) - 2\times 4835\times 1397\times 5000\,x = 1397^2 (R^2 - 5000^2)$

ie. $x^2 (25328834) - 2 \times 4835 \times 1397 \times 5000\, x = 1397^2 (R^2 - 5000^2)$

$$\therefore \quad x^2 - \frac{2 \times 4835 \times 1397 \times 5000\, x}{25328834} = \frac{1397^2 (R^2 - 5000^2)}{25328834}$$

Converting $\frac{615}{2525}$ into a continued fraction we have

$\frac{1}{3+}\frac{1}{1+}\frac{1}{3} = \frac{4}{15}$ so that the equation could be written as

$$x^2 - 2 \times 5000 \frac{\times 4}{15} = \frac{1397^2 (R^2 - 5000^2)}{25328834}$$

Here $\frac{5000 \times 4}{15}$ is symbolized as Ādya so that we have

$x^2 - 2$ Ādya $x =$

$$5000^2 \times \frac{16}{225} - \frac{5000^2 \times 1397^2}{2532883} + \frac{1397^2 \times 3438^2}{2532883}$$

$$\left(\frac{16}{225} \quad \frac{1397^2}{2532883}\right) + \frac{1397^2 \times 3438^2}{2532883}$$

Here $\frac{16}{225} - \frac{1397^2}{2532883}$ is approximated to $\frac{-2}{337}$

and $\frac{1397^2 \times 3438^2}{2532883}$ is approximated to 910678 so that we have

$x =$ Ādya $\pm \sqrt{910678 - \frac{2\, s^2}{337}}$ where s is the given sum.

Since the positive sign of the radical is invalid because H sin $\delta \not> R$, so the negative sign is taken.

Verse 102. In a place where $s = 5''$, the sum of H sin δ, S. S., Taddhṛti, Kujyā and Agrā is 6500; find them individually *oh*, mathematician, if thou art adept in understanding the sphere and dealing with the latitudinal triangles.

Verse 103. Answer to the problem above.

Assuming H sin δ to be equal to 12 s and computing the various quantities cited; take their sum. Then by rule of three "If for this sum got, the individual magnitudes are such and such what will they be for the given sum" each can be had.

Comm. The cited magnitudes are respectively H sin δ, $\frac{\text{R H sin } \delta}{\text{H sin } \phi}$, $\frac{\text{R}^2 \text{ H sin } \delta}{\text{H sin } \phi \text{ H sin } \phi}$, $\frac{\text{H sin } \delta \text{ H sin } \phi}{\text{H sin } \varphi}$ and $\frac{\text{R H sin } \delta}{\text{H cos } \phi}$ which are all proportional to H sin δ, ϕ being given through 's'. With this idea of proportionality at the back of his mind, Bhāskara sets this ingenious question, and gives an easy way of solving it by assuming H sin δ to be $5 \times 12 = 60$, so that the others can be got rationally.

With this H sin δ, S. S. $\frac{3438 \times 60}{3438 \times 5/13} = 156$, Taddhṛti

$$\frac{3438^2 \times 60}{3438 \times \frac{5}{13} \times 3438 \times \frac{12}{13}} = 169; \text{ Kujyā} = \frac{60 \times 5}{13 \times 12/13} = 25$$

$$\frac{\times 60}{3438 \times 12/13} = 65$$

The sum of these is 475. So, by the rule of three mentioned above, H sin δ = 1200, S. S. = 3120. Taddhṛti = 3380, Kujyā = 500 and Agrā = 1300.

Or alternatively given $s = 5$, $k = 13$ so that H sin ϕ =

$$\phi = \frac{3438 \times 12}{13}.$$

the various magnitudes are H sin δ, $\frac{\text{H sin } \delta \times 13}{5}$,

$\frac{\text{H sin } \delta \times 13^2}{60}$, $\frac{\text{H sin } \delta \times 5}{12}$ and $\frac{\text{H sin } \delta \times 13}{12}$.

The sum of these is H sin δ $\left(1 + \frac{13}{5} + \frac{169}{60} + \frac{5}{12} + \frac{13}{12}\right)$

$$\text{H sin } \delta \left(\frac{60 + 156 + 169 + 25 + 65}{60}\right) = \frac{475}{60} \text{ H sin } \delta = \frac{95}{12}$$

H sin δ = 9500 $\therefore$ H sin δ = 1200 from which by substitution the remaining magnitudes could be obtained.

Verse 104. If the sum of Agrā, H sin δ and Kujyā be 2000 find them individually, oh ! mathematician if thou be an adept in the geometry of the sphere and computation.

Comm. Here the quantities are respectively $\frac{\text{R H sin } \delta}{\text{H cos } \varphi}$, H sin δ and $\frac{\text{H sin } \delta \text{ H sin } \varphi}{\text{H cos } \varphi}$ so that their sum is H sin $\delta\left(1 + \frac{\text{R}}{\text{H cos } \varphi} + \frac{\text{H sin } \varphi}{\text{H cos } \varphi}\right)$

$$= \frac{\text{H sin } \delta \ (\text{H sin } \varphi + \text{H cos } \varphi + \text{R})}{\text{H cos } \varphi}$$

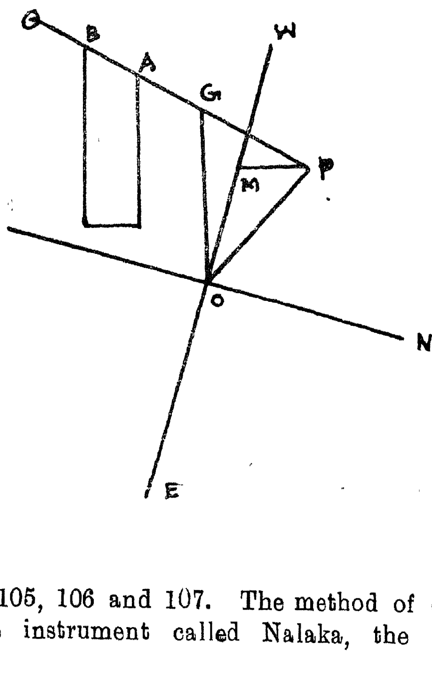
Here also we are to presume $s = 5$ so that the above sum is $\frac{\text{H sin } \delta \ (s + 12 + k)}{12}$ (by proportion of the first and second latitudinal triangles) = $\frac{\text{H sin } \delta \ (5 + 12 + 13)}{12}$ =

$\frac{5}{2}$ H sin δ = 2000 $\therefore$ H sin δ = 800. Substituting this value in the above formula, Agrā

$$= \frac{\text{RH sin } \delta}{\cos \varphi} = \frac{3438 \times 800 \times 13}{3438 \times 12} = \frac{10400}{12} = 866\text{-}40;$$

$$\text{Kujyā} = \frac{\text{H sin } \delta \text{ H sin } \varphi}{\text{H cos } \varphi} = \frac{800 \times 5}{12} = \frac{4000}{12}$$

OG = Gnomon; OP = Shadow of the gnomon; PM = the Bhuja drawn from the extremity of the shadow P perpendicular on the East-west line; OM = Koti of the shadow extending along the East-west line. AB is the Nalaka placed along the Chayakarṇa PG. The eye is placed at A and the planet ⊙ is visible through the tube of the Nalaka AB.



Verses 105, 106 and 107. The method of observing through the instrument called Nalaka, the planetary position.

On a horizontal plane mark a point and through it draw the East-west line and also the North-south; if the planet is in the East mark off the computed Koti of the shadow towards on the East-west line; if the planet is in the Western hemisphere, mark this Koti towards the East. From the extremity of the Koti mark the computed Bhuja perpendicular to the East-west line and draw the computed shadow from the point so as to form a right-angled triangle with the Bhuja and Koti. Extend a thread from the point of intersection of the bhuja and shadow to meet the gnomon's top so as to form the Chāyākarṇa or the hypotenuse of the right-angled triangle of which the other sides are the gnomon and the shadow. Along this thread place the Nalaka such that the lower extremity of the Nalaka coincides with the eye. Seeing through the Nalaka, the planet is to be seen. I shall tell how the planet could be seen in water as well.

Comm. The Nalaka is a simple tube formed generally of bamboo. The purpose of this is to verify the correctness of the computation of the shadow and its bhuja. If the computation is wrong the planet will not be seen in that direction. It might be asked how the shadow and bhuja are pertinent with respect to a planet, whose shadow cannot be observed as that of the Sun. True, but the computation of the shadow and bhuja are done as will be done with respect to the Sun, knowing the declination etc. as in the case of the Sun. Computation does not depend on the observation of the actual shadow. Computing the magnitudes of the Bhuja and Koti, the direction of the Chāyākarṇa points to the planet in the sky.

Verse 108. Observing the planet through the Nalaka in water.

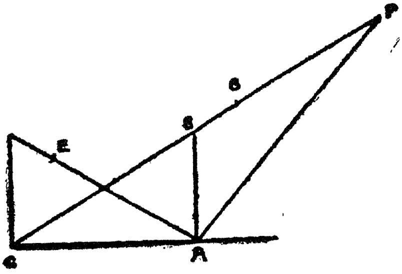


Fig. 63

Place the S'anku at the point of intersection of the Bhuja and shadow and holding the Nalaka along the join of the top of S'anku and the point, the planet could be seen in a basin of water placed at the point.

Comm. Let P be the planet casting the shadow AC of the gnomon AB. C the extremity of the shadow is the point of intersection of the shadow and the Bhuja. Though we have shown the gnomon in the position AB, it need not have been placed there in as much as we have the computed magnitudes of the shadow, the Bhuja and the Koti. Now we are directed to place the S'anku actually at C the point of intersection of the shadow and the Bhuja. Thus CD is the S'anku. Since CD=AB and and both are vertical evidently $\triangle s$ DBA and DCB are congruent. Hence $\widehat{DCB} = \widehat{DAB}$. But $\widehat{DCB}$ = zenith-distance of the planet and as such is equal to $\widehat{BAD}$ (also the zenith-distance of the planet) $\therefore$ $\widehat{DAB} = \widehat{BAP}$. Hence if a tray of water is placed at A, the planet will be visible as seen through DE, the Nalaka since the angle DAB is the angle of incidence and $\widehat{BAP}$ the angle of reflection are equal.

Verse 109. The planet is to be shown to the king, who has an eye of appreciation for the same, either directly (as shown in fig. 62) in the sky or through water as shown in the fig. 63, having finished the preliminaries indicated.

Comm. Clear.

End of the Tripras'nādhyāya.

PARVASAMBHAVĀDHIKĀRA

Investigation into the occurence of an eclipse

Verses 1–2. Multiply the number of years that have elapsed from the beginning of the Kaliyuga by twelve and add the number of months elapsed from the beginning of the luni-solar year. Let the result be x. Then add

$\frac{\quad}{65}$ to x. Let the result by y. Then the longitude of what is called Sapāta-Sūrya or the longitude of the Sun with respect to a node will be x Ras'is +

$\frac{(2y + 503)\ (1 + \frac{1}{169})}{3 \times 30}$ Ras'is. If this longitude be less than 14°, then a lunar eclipse is likely to occur.

Comm. The first operation indicated above in directing x to be added to $\frac{2x\ (1 - \frac{1}{898}}{65}$ is intended to obtain the lunations that have elapsed from the beginning of the Kaliyuga. In this behalf we are asked to multiply the elapsed years by twelve to get the number of solar months. Here there is one subtlety to be noticed. The years that have elapsed are not entirely solar. In fact the years reckoned according to the luni-solar system were all originally luni-solar; but according to the convention of intercalary months, they were rendered solar upto the point of the latest intercalation, for, solar months plus intercalary months are equal to the elapsed lunations. From the moment of the end of the latest intercalary month, the subsequent years or year or fraction thereof would be luni-solar only. Nonetheless, no difference will be there in the computed Adhikamāsas in adding a few lunar months to the solar and taking them all to be solar. The maximum error committed in so doing will be of the order of (no. of days in a solar month minus no.

of days in a lunar month) multiplied by $36 \times \frac{2}{65} \times \frac{1}{30}$ of an adhikamāsa, assuming that an adhikamāsa would occur at the latest in 36 solar months. (In fact, an adhikamāsa would occur on the average in $32\frac{1}{2}$ solar months, but we have taken 36 roughly as the maximum figure in as much as the occurence of the Adhikamāsa might be belated on account of the convention stipulated). Thus the error would be $36 \times 2 \times \frac{2}{65} \times \frac{1}{30} = \frac{1}{13}$th of an adhikamāsa at the maximum. Hence, we are directed not only to construe that all the years elapsed to be solar but also the subsequent lunations of the current luni-solar year also to be solar months. Thus getting the number of elapsed months from the beginning of the Kaliyuga, the computation of the Adhikamāsas is formulated as follows. If in the course of 51840000 solar months of the Yuga there be 1593300 Adhikamāsas then during the elapsed solar months x, what is the number of elapsed Adhikamāsas? The result is

$$\frac{x \times 1593300}{51840000} = \frac{x \times 1593300}{\frac{796650}{\frac{51840000}{796650}}}$$

$= \frac{2 \times x}{65\text{-}4\text{-}21}$. Since Bhāskara knows that there will be two Adhikamāsas roughly in 65 solar months, he performed the above operation. This shows that for every 65 solar months roughly there occur two Adhikamāsas or more accurately a little less than two Adhikamās. So, taking, in the first instance 2/65 as the ratio of Adhikamāsas to the number of solar months, Bhāskara tries to find as to what quantity is to be subtracted from 2. That is found as follows. If there be A adhikamāsas in s solar months what will be the number of Adhikamāsas in x solar months? The result is $\frac{A x}{s}$. Again if there be two Adhikamāsas roughly in 65 solar months, how many will be there in x solar months? The answer is $\frac{2x}{65}$. But we have seen about that the

accurate number should be $\frac{2x}{65} - \lambda$ ie. a little less than $\frac{2x}{65}$. The question is now to find the value of λ. So, equating $\frac{2x}{65} - \lambda$ to $\frac{Ax}{s}$, $\lambda = \frac{2x}{65} - \frac{Ax}{s} = x\left(\frac{2}{65} - \frac{A}{s}\right)$
$= x\left(\frac{2s - 65A}{65s}\right)$. Substituting for $2s - 65A$ namely

$$2 \times 51840000 - 65 \times 1593300 = 115500$$

$$\lambda = \frac{x \times 115500}{65 \times 51840000} = \frac{x \times 2 \times 57750}{65 \times 51840000} = \frac{2x}{65} \times \frac{1}{\frac{51840000}{57750}}$$

$$= \frac{2x}{65 \times 898} \quad \therefore \quad \frac{Ax}{s} = \frac{2x}{65} - \lambda = \frac{2x}{65} - \frac{2x}{65 \times 898}$$

$$= \frac{2x}{65}\left(1 - \frac{1}{898}\right) \text{ as given.}$$

The procedure, adopted as above, is in a way a short cut in Hindu Astronomy to obtaining a convenient convergent to a continued fraction. Let us use the method of continued fractions; the number of Adhikamāsas in x solar months is $\frac{Ax}{s}$ ie. $x \times A/s =$

$\frac{x \times 1593300}{51840000} = \frac{x \times 5311}{172800}$. Converting $\frac{172800}{5311}$ into a continued fraction we have

$32 + \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{6+}\,\frac{1}{2} + \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{18+}\,\frac{1}{4}$ to which $\frac{65}{2}$ is a convergent

but a good convergent is $\frac{245}{69}$. As this good convergent is unwieldy, Bhāskara used 2/65 and made amends for the roughness introduced by adopting it. Wherever a convenient convergent is not available, an easy and rough convergent is used and amends will be made for the roughness resulting as follows. Let $\frac{M}{N}$ be a fraction to which $\frac{m}{n}$ is a convergent having small numbers as numerator and denominator, so that $\frac{M}{N}$ is taken to be equal to $\frac{m}{n}\left(1+\frac{1}{\lambda}\right)$.

Thus $\frac{M}{N}=\frac{m}{n}\left(1+\frac{1}{\lambda}\right)$ or $Mn = Nm\left(1+\frac{1}{\lambda}\right)$

$\therefore \quad Mn - Nm = \frac{Nm}{\lambda}$ or $\lambda = \frac{Nm}{Mn - Nm}$. In the present case M is the number of Adhikamāsas, and N the number of solar months. $\frac{m}{n}$ if taken to be $\frac{2}{65}$

$$\therefore \quad \text{In this case } \lambda = \frac{-2 \times \text{Solar months}}{65 \times \text{Adikamāsas} - 2 \times \text{Solar months}}$$

which is indicated in the commentary by Bhāskara.

In this context, it may be mentioned that a Karaṇagrantha named Nārasimha based upon Sūryasiddhānta (A Karaṇagrantha is a manual according which the Hindu calendar is computed with easy numbers without undergoing the laborious process indicated in the treatises called Siddhāntas like the present Siddhānta S'iromani. In these Karaṇagranthas, instead of taking the beginning of the Kalpa or Mahāyuga or the Yuga, as the epoch, a recent date ie. the date of the author of the Karaṇagrantha is taken as the epoch, and processes using approximations are adopted for the sake of ease. Naturally therefore these Karaṇas (as they are also called) get easily obsolete within the course of a few hnudreds of years so that a fresh Karaṇa is called for preparation, if the calculations were to accord with the Siddhāntas which those Karaṇas profess to follow. In fact, the present Karaṇa of Nārasimha written in 1333 S'aka year ie. in 1411 A.D. declares that a previous Karaṇa named Tithicakra reported to have been written by one Mallikārjuna Suri grew obsoleto and so, a fresh Karaṇa is being written to accord with the original Sūrya Siddhānta, (Vide verses I, II of Nārasimha) "तिथिचक्रं यत् प्रणीतं मल्लिकार्जुनसूरिणा, कालेन महता तस्मिन् खिलीभूते तदादरात्, नौपुरी सिङ्ग्यार्यस्य नरसिहेन सूनुना, एतदेव स्फुटतरं क्रियते सौरसम्मतम्" This Karaṇa takes 3/98 instead of 2/65 as a convergent in computing the Adhikamāsas. In this case the continued fraction becomes $32 + \frac{1}{1+}\frac{1}{1+}\frac{1}{1+}$. So that the convergents are $\frac{32}{1}$, $\frac{33}{1}$, $\frac{65}{2}$, $\frac{98}{3}$. Taking 98/3 as a convergent, let us see how it was sought to make amends for the roughness of the convergent. As per the Sūrya-siddhānta the number of Adhikamāsas during the course of 51840000 solar months of a Yuga are 1593336 as against 1593300 prescribed by Bhāskara. Hence putting

$$\frac{1593336}{51840000} = \frac{3}{98}\left(1 + \frac{1}{\lambda}\right)$$

ie. $\frac{66389}{2160000} = \frac{3}{98}\left(1 + \frac{1}{\lambda}\right)$, $\lambda = \frac{3 \times 2160000}{66389 \times 98 - 3 \times 2160000}$

$= 248$ very approximately. So Nārasimha adopts for the Adhikamāsas instead of $\frac{2}{65}\left(1 - \frac{1}{898}\right)$, the formula 3/98

Having added the Adhikamāsas so obtained to the solar months we have the lunations that have elapsed from the beginning of the Kaliyuga. Then the next procedure indicated is as follows. In an eclipse solar or lunar, the celestial latitude of the Moon has to be less than a particular limit for the occurence of an eclipse. Thus in the case of a solar eclipse the celestial latitude of the Moon must be less than 32′, whereas in the case of a lunar eclipse it is to be less than 56′. Of course, for a solar eclipse to be possible anywhere on the earth, not for a given place, the limit is far higher as given in texts of modern astronomy namely $p_m + s + m - p_s$ where p_m is the horizontal parallax of the Moon, p_s that of the and s and m the angular semidiameters of the Sun and the Moon. (This formula we shall see later). This higher limit comes to 88.5′. The limit of 56′ for the occurence of a lunar eclipse is the value of

$$p_m + p_s - s + m \text{ as we shall see later.}$$

The latitude of 56′ of the Moon arises out of a longitude of 12° of the Moon with respect to a node, whereas the latitude of 32′ arises out of a longitude of 7° with respect to the node. Since at an eclipse solar or lunar, the longitude of the Moon with respect to a node, is the same as the longitude of the Sun with respect to the same or opposite node, the latter must be 12° for the occurence of a lunar eclipse. But as the difference between the mean and true Suns is about 2°, the longitude is stipulated as 14°. In other words, for the oecurence of a lunar eclipse, the longitude of the Sun on the full-Moon day with respect to the nearer node shall be less than 14°. To compute this longitude of the Sun with respect to the nearer node on a full-Moon day, we are given the subsequent procedure indicated in the verse. In 53433300000 lunations of the Kalpa, the sum of the sidereal revolutions of the Sun and the Node (Rāhu) (Sum because Rāhu has a retrograde motion) is equal to 455231168 which is equal to 455231168 × 12 = 54627734016 Ras′is. Then in one lunation what will be the increase of the longitude with respect to the Node? The result is

$$\frac{54627734016}{53433300000} = 1 \text{ Ras'i} + \frac{3583302048^\circ}{5343330000}\left(= \frac{74652126}{111319375}\right)$$

dividing by 48 both the numerator and denominator. Taking the first two digits in the numerator and denominator of the fraction the fraction is approximately equal to $\frac{36}{54}$ or 2/3. Taking this as a convergent we make amends for the roughness as follows.

$$\frac{74652126}{111319375} = \frac{2}{3}\left(1 + \frac{1}{\lambda}\right) \therefore \lambda = \frac{2 \times 111319375}{3 \times 74652126 - 2 \times 111319375}$$

$= \frac{222638750}{1317628} = 169$ approximately. Hence the increase of the Sun's longitude with respect to a node is

1 Rāśi $+ \frac{2}{3}\left(1 + \frac{1}{169}\right)^{\circ}$ I. In the beginning of the Kaliyuga, the longitude of the node was 5 Rāśis—3°—13′ and the arc moved by the Sun with respect to the node during the course of half a lunation is 0—15—20, so that their sum is 5—18—33. Here we have added for half a lunation because the context is a lunar eclipse and the beginning of the Kaliyuga was a New Moon day. Also, at the beginning of the Kali, the Mean Sun being at the zero-point of the zodiac, the negative longitude of the node only is the longitude of the Sun with respect to the node. Hence we have to add the above longitude of 5—18—33 to the longitude obtained through the above formulation I, which means 168°—33′ is to be added to $\frac{2x}{3}\left(1 + \frac{1}{169}\right)$ where x is the elapsed number of lunations. Taking 168°—33′ as nearly equal to 168°—40′, $\frac{2x}{3}\left(1 + \frac{1}{169}\right) + 168\frac{2}{3}^{\circ}$

$$= \frac{2x}{3}\left(1 + \frac{1}{169}\right) + \frac{506}{3} = \frac{2x}{3}\left(1 + \frac{1}{169}\right) + \frac{503}{3}\left(1 + \frac{1}{169}\right)$$

approximately $= \frac{(2x + 503)}{3}(1 + 1/169)$ as formulated. Thus for x lunations, the longitude of the Sun with respect to the node is x Rāśis $+ \frac{(2x + 503)}{3}\left(1 + \frac{1}{169}\right)^{\circ}$. If this longitude falls short of 14°, we could except a lunar eclipse.

Latter half of verse 3 and *verses* 4, 5. Particularity with respect to a solar eclipse.

Add half a Rāśi to the longitude previously obtained; find out on which side the Sun lies, north or south; compute the longitude of the Sun from the number of days elapsed after the Samkramaṇa day (ie. the day on which the Sun has left one Rās'i and entered another); obtain the hour-angle in nādis of the Sun at the ending moment of the Amāvāsyā ie. at the moment of New Moon; add or subtract one-fourth thereof in Rās'is from the position of the Sun according as the Sun is in the Western or Eastern hemisphere; then finding the declination of that point and from the sum or difference of the declination and latitude of the place, obtain the zenith-distance of the culminating point of the ecliptic; taking that point to be roughly the Vitribha ie. the point of the ecliptic which is 90° behind the Sun on the ecliptic, find one-sixth of the zenith-distance; taking the sum or difference of the result and the longitude of the Sun with respect to the node (got in the beginning by adding half a Rās'i to his position at full-Moon) if the result happens to fall short of 7°, then we could expect a solar eclipse.

If there be no eclipse at the current New Moon, then go on adding 1 Rās'i $-0^\circ -40' -15''$ to the longitude of the Sun with respect to the Node (which will be his longitude for the moment of the next New Moon) and repeating the procedure indicated, the occurence of an eclipse or otherwise could be known. If occurence be indicated then compute the actual positions of the Sun, Moon and Rāhu and following the procedure to be indicated in the chapter on solar eclipses, the moment of the occurence of the eclipse and other relevant details could be computed.

Comm. In the case of a lunar eclipse, the Manaikyārdha ie. half the sum of the diameters of the eclipsing and eclipsed bodies (namely the cross-section of the Earth's shadow at the lunar orbit and the Moon) is 56′. This is the maximum limit to the celestial latitude of the Moon if an eclipse were to occur, and this latitude will be there if the longitude of the Moon with respect to the nearer node is 12°. Since a lunar eclipse occurs at the moment of a full Moon, the distance of the Sun then from the opposite node should be also 12° for the occurence of an eclipse. Since it is customary to check the occurence of an eclipse through Sapātasūrya ie. longitude of the Sun with respect to the node (the prefix Sa is to signify that the sum of the Sun's longitude and that of the node should be taken, as the longitude of a node is measured in the opposite direction from the zero-point of the ecliptic) it is stipulated that the Sapātasūrya should be 12° for the occurence of a lunar eclipse. But, as the True Sun might differ from the Mean by about 2°, and as we are concerned with the True Sun only, the limit is increased by 2°, so that a lunar eclipse may occur if the Sapātasūrya happens to be less than 14°. Thus there is no more complication with respect to the occurence of a lunar eclipse than requiring the longitude of the Sapātasūrya to be less than 14° for the occurence of a lunar eclipse. If this condition be satisfied, there will be an eclipse and that will be visible at all places, where there is night, since a body in shadow will not be seen from any place whatsoever.

But, there is a complication with respect to the occurence of a solar eclipse namely that it is not a question of the Sun entering a shadow. The Sun could never be shadowed. A solar eclipse occurs when the disc. of the Moon comes in between the Sun and an observer and obstructs a vision of the Sun's disc. The Moon being very near us compared with the Sun, its coming in between the Sun and an observer may well be compared with a cloud obstructing the vision of the Sun. Just as, when a cloud obstructs the vision of the Sun for an observer, it could not do so with respect to another who is situated at a distance, so also, if the Moon effects a solar eclipse for a particular place, it could not do so for all places. This is said to be due to parallax or Lambana as it is called (Refer fig. 64). It is so called because, when there is an eclipse of the Sun for an imaginary observer at the centre C of the Earth, the Moon intersecting in the line of sight to

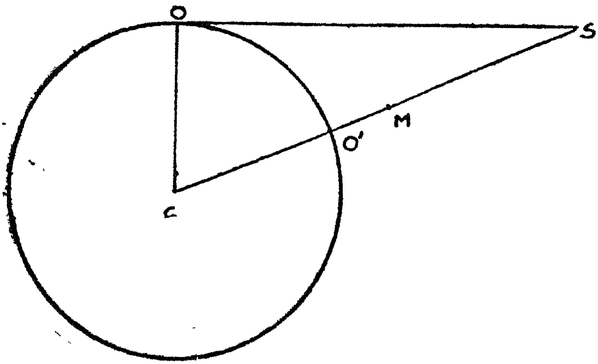


Fig. 64

the Sun, for an observer 'O' on the surface of the Earth, the Moon is not in the line of sight namely OS but hangs down that line (लम्बते अनेनेति लम्बनम् = That phenomenon by which the Moon hangs down the line of sight). Hence it is not sufficient to say that the Sapātasūrya has a longitude of 7° to conclude the occurence of a solar eclipse for a place. If the Sapātasūrya be less than 7°, certainly there will be a solar eclipse for some place on the earth but not for all places. So, to decide whether there will be a solar eclipse for a given place, we are to take into account the phenomenon of parallax. At every New Moon, the longitudes of the Sun and the Moon will be no doubt equal; yet the Moon may not obstruct a vision of the Sun, not being situated in the ecliptic plane. He may be above the ecliptic plane or below it and if he be within 32′ from the plane, a part of the Moon's globe may hide a part of the Sun's from the vision of certain observers who are situated about the point o′ of fig. 64. But suppose an observer is at O. For him. there is no eclipse at all as could be seen from the figure, As the observer moves away and away from O′ towards O, the effect of parallax will be greater and greater in longitude, whereas, as the observer moves away and away from O′ towards O_1 (where O_1 is the geocentric pole of the circle

(c) shown in the figure) along the circle of intersection of the Earth with a plane through CO′ perpendicular to the plane of the paper, where O_1 is a point on the earth such that $\widehat{SCO}=90°$ the effect of parallax will be greater and greater in latitude. In other words the parallax has both an effect in longitude as well as in latitude. When it has an effect in longitude only it is called Lambana, whereas, when it has an effect in latitude, it is called Nati. (Thus the translation of 'parallax' as Lambana alone is not fully correct, though at times the parallax may have its complete effect in longitude only or in latitude only). For the observer who moves in the ecliptic plane only as the one moving from O′ towards O, the parallax will have its entire effect in longitude only and for the observer moving in the perpendicular plane from O′ to O_1 mentioned before, the parallax will have its entire effect in latitude only. For observes other than the two above, it will have effect both in longitude and latitude also. When parallax effects longitude the time of conjunction is preponed or postponed, whereas when it effects latitude, the latitude of the Moon appears to have increased or decreased. When it increases, no eclipse occurs and when it decreases an eclipse does occur. At O′, the latitude will be exactly what has been computed; at the point of intersection O″ of the join of the centres of the Sun and Moon with the surface of the Earth, parallax nullifies the latitude and on the great circle O′ O″ there will be parallax in latitude only effecting the magnitute of the latitude.

It will be seen that for the point O″, the Sun and the Moon are in the zenith, so that neither will suffer from parallax. For the point O the Sun will be on the horizon and the Moon being depressed below the horizon though he is in geocentric conjunction the parallax in longitude or lambana is maximum and the occurence of the New Moon had already elapsed 4 nādis ago.

it will be seen that at the points O_1 and O_2 where O, is the other geocentric pole of the circle (c) drawn there cannot be an eclipse, the latitude being incresaed (as per Hindu astronomy) by 48′ − 46″. In fact, there will be eclipse for the places on either side of O″ (the point of intersection of the join of the centres of the Sun and Moon with the Earth's surface) to such a distance as will increase the latitude to 32′ only. For the other places on O′ O″ beyond these points, the latitude of the Moon exceeds this limit and so there will be no eclipse. Further clarification of Lambana and Nati will be given later.

The above analysis underlies our investigation for the occurence of a solar eclipse. Sapātasūrya might be less than 7°, but it does not mean that every place will enjoy an eclipse. So, for the place concerned, we have to see that, even after taking into account the parallax in latitude ie. in Nati, still the latitude will be less than 32′. To obtain this parallax in latitude, the method adopted is to find it at the point called Vithribha, (nonagesimal) ie. the point which is behind the Lagna the rising point of the Ecliptic by 90°; for, as we shall see in the Chapter on Solar eclipses, the parallax in latitude at the Vitribha will be equal to the parallax in latitude at any point of the ecliptic. In other words, wherever be the Sun and Moon on the ecliptic (Moon also being very near the node may be taken roughly to lie on the ecliptic) to compute the amount by which the latitude is increased, we compute it for the Vitribha, and this will hold good for the arbitrary position of the Moon, for, there also the latitude will be increased by the same amount. The procedure given in verse 4 is to locate the Vitribha from the position of the Sun and to find its zenith-distance to compute the Nati; or rather, it is to locate the culminating point and taking it roughly to be Vitribha, to compute the influence of Nati on the latitude or S'ara. If it were only to find the Vitribha, it could be computed from the lagna of the moment of New Moon. Computation of the zenith-distance of the Vitribha is a little cumbrous, so that, for brevity, it is sought to compute the culminating point, obtain its declination and thereby its zenith-distance which could be taken to be the zenith-distance of the Vitribha also, from which the Nati is calculated.

We are directed to obtain first the Nata or the hour angle of the Sun for the moment of New Moon. We know on that particular New Moon day how long Amāvāsya will last after Sun-rise ie. we know when the actual moment of New Moon occurs on that day. We also know the duration of day time on that day so that subtracting the time of occurence of the New Moon after Sun-rise from half the duration of day, we obtain the hour angle of the Sun (Nata) in nādis. Now the Moon's longitude is effected by parallax, the effect being depression of the Moon. It is roughly estimated that the hour angle expressed in nādis is increased by $\frac{1}{4}$ of its value on account of this. Strictly speaking the effect of parallax is far more on the position of the Moon than on the Sun. But the Hindu procedure apparently treats the Sun alone for parallax. The reason is that at the moment of geocentric conjunction of the Sun and the Moon, when we consider the combined effect of parallax on the Moon and the Sun at once, for a given place, we may as well compute the relative position of the Sun effected by parallax. Let the hour-angle of the Sun in nādis be x. Then effected hour-angle will be $x\,(1 + \frac{1}{4}) = \frac{5x}{4}$ nādis. But each Rāśi being taken roughly to rise in 5 nādis, the hour-angle in Rāśis will be $5x/4 \div 5$ Rāśis $x/4$. Hence we are directed to divide the hour-angle of the Sun in nādis to divide by 4. This $x/4$ being substracted from the longitude of the Sun, we get the longitude of the culminating point. Then we are directed to obtain the declination of the culminating point from the formula $H \sin \delta = H \sin \omega \times H \sin \lambda \div R$, This declination of the culminating point being known, and the latitude being known, its meridian zenith-distance could be got. Take that to be roughly the zenith-distance of the Vitribha. Then an approximate estimate of the effect in latitude is obtained as follows. Let the zenith-distance of the Vitribha be x. Then if we have for H sin z = R, the maximum effect of $48' - 46''$ in the latitude, what shall we have for H sin 45° ? The result is

$$\frac{\text{H sin } 45 \times 48' - 46''}{3438} = \frac{2431}{3438} \times 48' - 46'' = 34' - 30''.$$

Then again another approximate estimate is made as follows. Let the Sapātasūrya (Sapātasūrya = longitude of the Sun or what is the same of the Moon with respect to the Node) be λ. Then for $\lambda = 15°$, we have a latitude of $70'$. That being so, for a variation of $34' - 30''$ in the latitude, what will be the corresponding variation in the longitude λ ?

The result is $= \frac{69}{2} \times \frac{15}{70} = \frac{207}{28} = 7° \frac{11}{28}$ which is roughly one-sixth of 45°. Hence if the zenith-distance of the culminating point taken to be the Vitribha be 45°' the variation in the Sapāta-sūrya (= Sapāta chandra) will be one-sixth thereof. Hence we are asked to increase or decrease as the may be, the Sapāta sūrya by 1/6th. If the resulting Sapātasūrya be less than 7°. there may be an eclipse. When the lunar orbit lies north of the ecliptic, the culminating point of ecliptic being south of the zenith, the latitude of the Moon is decreased by parallax, so that, we have to decrease the Sapātasūrya; whereas when the lunar orbit is then south of the ecliptic, the effect of parallax is to increase the latitude and consequently, we have to increase the Sapātasūrya. Or again when the culminating point of the ecliptic is north of the zenith and the lunar orbit is north of the ecliptic, parallax appears to increase so that we have to increase the Sapātasūrya; when at that moment the lunar orbit is south of the ecliptic, parallax appears to decrease the latitude so that we have to decrease the Sapātasūrya.

The proportion that 70′ of latitude correspond to 15° of Sapāta-chandra is due to the formula

$\frac{H \sin 15° \times H \sin 4\frac{1}{2}}{R} = H \sin \beta$. Using logarithms,

$\log \sin \beta = 9{\cdot}4130 + 8{\cdot}8946 = 8{\cdot}3076$ so that $\beta = 1° - 10' = 70'$. This proportion could be used because the Moon is within 15° of the Node. Hence when we are investigating the occurence of a solar eclipse, application of rule of three is not unjustified or crude.

Herein, Bhāskara assumed the zenith-distance of the nonagesimal to be round about 45° and drew a conclusion that the effect in the longitude on account of parallax in latitude is one-sixth of the zenith-distance. Instead if the zenith-distance be assumed to be z, the result would be $\frac{48' - 43'' \times \sin z \times 15}{70} = \frac{21}{2} \sin z$ which may be taken as a better approximation. If, on the other hand the modern value of 57' of the lunar parallax be taken, the result would be $\frac{57 \sin z \times 15}{70} = 12 \sin z$ approximately which is a better value.

In this calculation, it is better to compute the zenith-distance of the nonagesinal using modern methods instead of assuming the nonagesimal to be on the meridian.

LUNAR ECLIPSES

Verse 1. The ritualistic purpose served at the time of an eclipse.

Scholars of Smṛtis and purāṇas declare that prayer, charity or offerings to gods made in fire at the moment of an eclipse conduce to much spirituality. Hence I give hereunder the methods of computing the moment of an eclipse (lunar or solar) in as much as such a knowledge apart from its religious importance, is also wrought with a beautiful mathematical treatment.

Verse 2. The initial procedure to be adopted to compute an eclipse.

To know the occurence of a solar eclipse, find the exact moment of the New Moon, which is indicated by the equality of longitudes of the Sun and the Moon, and to know the occurence of a lunar eclipse compute the exact moment of the full Moon which is indicated by the fact that $M = S + 180°$ where M and S are the longitudes of the Moon and the Sun, agreeing in degrees, minutes and seconds, though differing in Rasis by six. Also compute the longitude of the Node (Rāhu) for the moment as directed.

Comm. Having ascertained the possibility for the occurence of a solar eclipse, we are directed to compute the positions of the Sun, the Moon and Rāhu for the day of the New Moon. The Sun and the Moon are to be rectified for corrections like Desāntara, Bhujāntara, Udayāntara etc. From the elongation of the Moon the tithi is to be computed and the method of successive approximations called Chālana Karma is to be used to obtain the exact moment of conjunction. This process of

Chālana is as follows. At first knowing the elongation of the Moon at Sunrise, by rule of three, using the then daily motions of the Sun and Moon, the moment of the conjunction is to be computed. Again the positions of the Sun and Moon are to be computed for that moment and also their daily motions are to be rectified for the moment. With these rectified daily motions and with the then positions of the Sun and the Moon, again the moment of conjunction is to be computed. Repeating the process till an invariable answer is reached, we have the exact moment of conjunction. For that moment, the position of the Rāhu is also to be calculated. Similar is the procedure for a lunar eclipse also. It is to be noted that the correction called Natakarma, which we formerly identified to be the correction for 'Astronomical Refraction' is also prescribed here as particularly mentioned by Bhāskara, in the commentary. (Vide verses 68, 69 Spaṣṭādhikāra).

Verse 3. The magnitndes of the orbits and the orbital radii of the Sun and Moon.

The distances of the centres of the globes of the Sun and the Moon from the centre of the Earth in yojanas are respectively 689377 and 51566.

Comm. We saw in the Kakshādhyāya of the first chapter as to how these distances were estimated. Some scholars pronounced these distances are parameters; but as per the modern estimate of the Earth's radius as compared with that of Bhāskara, (The method indicated by him in the Bhuparidhimānādhyāya of chapter I, is quite mathematical) a yojana equals five modern miles and with this correspondence, the distance of the Moon is very near the truth. So to say that the distances given above are mere parameters is wrong; also if one of the parameters gives a correct value of the quantity in question, the other also should; but because the latter does not, to call them parameters is merely meaningless.

It is intersating to note that in the commentary under this verse, Bhāskara says "If for a circumference of 3927, the diameter will be 1250, ..." This means that Bhāskara takes $\pi = \frac{3927}{1250} = 3{\cdot}1416$ which compares very well with the modern value 3·14159. The value of π adopted by Bhāskara in this, seems to have been taken from Lallācharya's Siṣyadhīvṛddhida, Chandragrahaṇādhikāra verse no. 3.

"शरयमाङ्गहता 625 भनवाग्निहृत्
ग्रहवृतिश्रवण: फलमुच्यते"

Verse 4. Computation of what is called the 'Kalākarṇa'.

The radius vector is to be computed even in the case of the Equation of centre as we did in the case of S'īghraphala. If it be 'K', $\frac{R^2}{2R - K}$ will be what is Kalākarna both in the case of the Sun, as well as the Moon.

Comm. While obtaining the Equation of centre, the formula used was $\frac{r \sin m}{R}$ whereas, strictly speaking, it should have been as in the case of the S'īghraphala, after effecting the so called 'Karṇānupāta' $\frac{r}{K}$ sinm. While trying to answer why this Karṇānupāta was not done there also, Brahmagupta gave such an answer as made Bhāskara exclaim 'यतो विचित्रा फलवासनाऽत्र' i.e. 'It is really curious in this respect.' Bhāskara was really a most rational type of astronomer, and one will not fail to appreciate his sense of rationality when he declares that (1) "अस्मिन् णितस्कन्धे उपपत्तिमानेव आगमः प्रमाणम्"; and when he was unable to adduce a proof he declares (2) 'उपलब्धिरेव वासना' meaning thereby (1) 'In this branch of science, we reckon only such an authority which has a proof behind it ie. which could be substantiated' and (2) 'There is no proof in this but accordance with observations alone has to be taken as a proof'.

Though in the case of formulating the equation of centre Karṇānupāta was not stipulated to simplify matters, as there was not much difference, the Equation of centre being generally small. It is to answer such contexts as this that Bhāskara said in the Golādhyāya

ie. 'If we in some particular context do not mention certain things it should not be condemned because in such contexts, (1) there is not much difference or (2) no useful purpose is served to a good extent or (3) it is too clear as does not require to be mentioned (4) the procedure implies a lot of cumbrous calculations and the result is after all negligible and (5) Mention will make the work on hand too unwieldy and brevity which is the soul of wit is to be sacrificed. In the present context this procedure of 'rectification of the Karṇa' is sought to improve matters. Bhāskara's words 'यदा ग्रहस्य कर्ण उत्पन्नः तदा कर्णो

:' seem really to imply that in the formula

$\frac{r}{R}$ H sin *m* for the equation of centre in the place of R, we are called upon to substitute really K. Though we had been in default for not doing so in the context of the Equation of centre, there is no reason why we should not make up the deficiency in this context. So, therefore, this rectification of Karṇa is stipulated here.

The procedure originally called for a rectification is that taking K to be R, we have to compute r and again taking the resulting K to be R, we have to compute r and so on repeating the process till an invariable value for K is obtained. This means that we should go on substituting for r, $\frac{r K}{R}$. Instead of following this laborious process of 'Asakṛt-Karma' ie. method of successive approximations, Bhāskara gives an alternative in the verse, whieh is as follows.

Let K be the value of the Karṇa, for a value r of r. Since we are directed to make this K as R, ie, we have to add R—K to K thus making it R, we add also R—K to R, to keep the relative position of R and K to be almost the same. In other words considering the fraction $\frac{K}{R}$, adding R−K to both the numerator and denominator we have $\frac{R}{2R-K}$ whieh means that for a radius 2 R −K, the Karṇa will be R; that being so for a Radius R what will be the Karṇa ? The result is $\frac{R^2}{2R-K}$ as given.

It will be noted that the above interpolative procedure is adopted as a short cut technique to the otherwise laborious process. The mathematical correctness of this procedure will be seen from the following analysis.

The problem is to change the Mandaparidhi to a radius K of the deferent by the formula (as indicated by Bhāskara in the course of the commentary) $r' = \frac{r K}{R}$ so that $\delta r = r^1 - r = \frac{r K}{R} - r = \frac{r(K-R)}{R}$ (1) Now we have $K^2 = R^2 + r^2 + 2 R r \cos m$ construing R and m as constants we have to find δ K for δr

Differentiating $2\,K\,\delta K = 2\,r\,\delta r + 2\,R \cos m\,\delta r$

ie. $\delta K = \dfrac{\delta r\,(r + R \cos m)}{K}$. But from fig. 65 (which is a portion of the epicyclic figure)

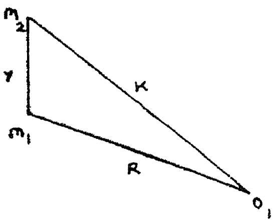


Fig. 65

$\hat{M}_1 = 180 - m$

$\hat{O}_1 = \theta$ = Mandaphala

$\hat{M}_2 = m - \theta$

$r = K \cos \overline{m - \theta} - R \cos m$ so that $r + R \cos m = K$ as $(m - \theta)$. Substituting in the above,

$$\delta K = \frac{\delta r \times K \cos \overline{m-\theta}}{K} = \delta r \cos \overline{m - \theta}$$

But δr from (1) is $\dfrac{r\,(K - R)}{R}$

$\therefore \quad \delta K = \dfrac{r\,(K-R)}{R} \cos \overline{m - \theta}$. But from the triangle of fig. 65. $K = R \cos \theta + r \cos \overline{m - \theta}$ so that $r \cos \overline{m - \theta} = K - R \cos \theta$. But θ being small $\cos \theta$ may be taken to be unity so that $r \cos \overline{m - \theta} = K - R$. Again substituting in the above

$\delta K = \dfrac{(K-R)^2}{R}$ (2) Now as per the formulation of Bhāskara

$$K^1 = \frac{R^2}{2\,R - K} = \frac{R^2}{R + R - K} = \frac{R}{1 + \frac{R - K}{R}} = \frac{R}{1 - \frac{(K - R)}{R}}$$

$$= R\left(1 - \frac{K - R}{R}\right)^{-1}$$

$\left|\dfrac{K-R}{R}\right| < 1$, expanding binomially,

$$K^1 = R\left(1 + \frac{K-R}{R} + \frac{(K-R)^2}{R^2}\right) = R + K - R + \frac{(K-R)^2}{R}$$

$$= K + \frac{(K-R)^2}{R} \quad \therefore \quad K^1 - K = \delta K = \frac{(K-R)^2}{R}$$ as found above.

Note. Bhāskara, having formulated this, appeals to 'Dhulīkarma' ie. arithmetical computation, for convincing those who may not be able to follow his logic. Here one may note also the wrong directive given by the Samśodhaka in the text. The proof furnished by us above gives a mathematical veracity to Bhāskara's formulation.

Verse 5. To rectify the Yōjanakarṇa or the spatial radius Vector.

The above Kalākarṇa multiplied by the Karṇa given in Yōjanas and divided by the Radius gives the rectified Yōjanakarṇa.

Comm. In the formula given above $\delta K = \frac{(K-R)^2}{R}$ which is in units of spatial minutes (on the scale of R=3438).

where K′ is the rectified Kalā-karṇa, and y the Yōjanakarṇa given in verse 3, gives the rectified Yōjanakarṇa.

Second half of verse 5. The spherical radii of the Sun and the Moon.

The spherical diameters of the Sun and the Moon are respectively 6522 and 480 Yojanas.

Comm. The word 'Bimba' is used to connote the spherical diameter. The diameter of Moon as given will be equal to 480×5=2400 miles in modern terms which is not far from truth. Once we accept that the method indicated by us in the Kakshādhyāya of chapter I was that followed by the ancient Hindu Astronomers to estimate the distance of the Moon, the spherical radius the horizontal parallax, the orbital radius pertaining to the Moon could all be deduced and the magnitudes so deduced accord with their average values in modern astronomy. • The magnitudes pertaining to the Sun however, should be deemed as parameters.

Verse 6. $e - \frac{(S-E)\,K_m}{K_s} = 2a$ where e = Earth's diameter, s = The Sun's diameter; Km = Moon's distance from the Earth's centre; Ks = The Sun's distance from the Earth's centre, and a = radius of the Earth's shadow cone at the lunar orbit.

Comm. In fig. 66, let CD be the radius of the Earth's shadow cone at the lunar orbit. Required to find the magnitude of CD. Triangles DEF and ESG are similar

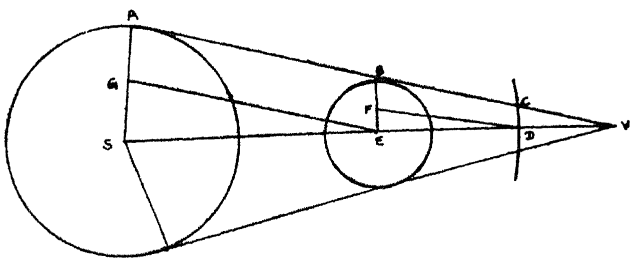


Fig. 66

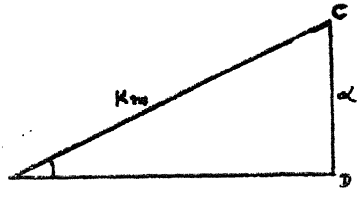


Fig. 67

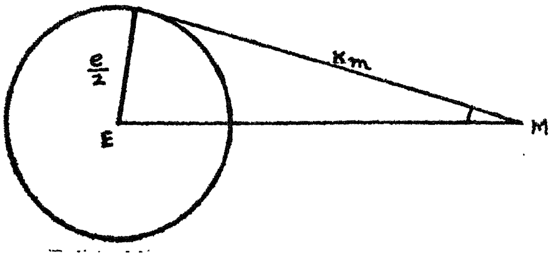
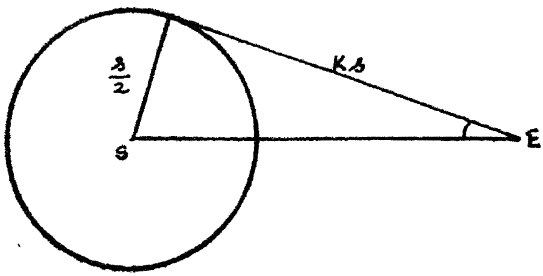


Fig. 68



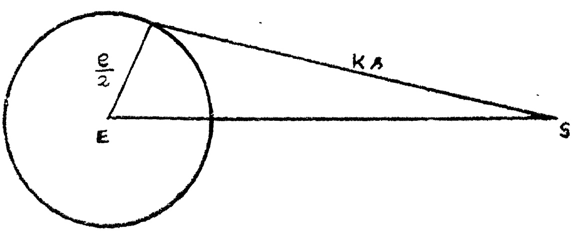


Fig. 70

where DF and EG are drawn parallel to VBA, the common tangent.

$$\therefore \quad \frac{EF}{DE} = \frac{SG}{ES} \quad \text{ie.} \quad \frac{\frac{1}{2}e - a}{Km} = \frac{\frac{1}{2}s - \frac{1}{2}e}{Ks}$$

where a = FB = CD required

$$\therefore \quad a = \tfrac{1}{2}\left\{e - \frac{K_m}{K_s}(s - e)\right\} \quad \text{I}$$ as formulated $2a$ being what is called Rāhu-Bimba or diameter of the Earth's shadow cone at the lunar orbit which is called Ku-bhā Vistṛti in the verse (Ku = Earth ; Bha = Shadow ; Vistṛti = diameter).

Note (1) We shall prove that this formula accords with the modern formula given for the radius of the shadow cone. Divide I throughout by 2 K_m, so that

$$\frac{a}{K_m} = e/2K_m - \frac{s-e)}{2\ K_s} \quad \text{II}$$

But from fig. 67. $a/K_m = \sin \hat{E} = \hat{E}$ = angular radius of the shadows cone expressed in radius. From fig. 68, $\frac{e}{2\ K_m}$ = Horizontal parallax of the Moon ; $\frac{s}{2\ K_s} = \sin \hat{E}$ $= \hat{E}$ = angular radius of the Sun expressed in radians from fig. 69 ; and $e/2K_s$ = (from fig. 70) Horizontal parallax of the Sun. Thus Equation II means $\rho = P - \sigma + P^1$ III where ρ = angular radius of the shadow cone, P = Horizontal parallax of the Moon ; P^1 = that of the Sun and σ = angular radius of the Sun.

Note (2) If we don't divide I by 2 K_m, we have the radius of the shadow-cone in Yojanas, substituting the values of e and s, K_m and K_s.

Note (3) It is worth hearing Bhāskara in his commentary under this verse. Observe the Sun's disc while rising on the day when his true motion is equal to his mean, with a compass composed of two rods hinged at one end and carrying a protractor at the other. We get the mean diameter of the Sun equal to 32′ - 31″ - 33‴. Similarly, observe the Moon's disc on a full-moon-day when his true motion equals the mean. It will be 32′ - 0″ - 9‴.

Note (4) Substituting the values of P, P^1 and σ in III $\rho = 52'\text{-}42'' + 3'\text{-}57'' - 16'\text{-}16'' = 40'\text{-}23''$ = Angular radius of the shadow-cone at the lunar orbit which almost accords with the modern value 41′-49″.

Note (5) One may wonder as to how, taking wrong values for s and K_s, such a correct value could be obtained for P. In equation II, α, K_m, e are all near the truth so that the terms effected are s/K and e/K_s; but both s and K_s being parameters s/K_s comes off alright, which is the angular radius of the Sun's disc which could be measured. The only vitiating term is e/K_s which is the horizontal parallax of the Sun which was overestimated unwittingly by a wrong supposition as indicated in the Kakshādhyāya. However e/K_s comes to be 3′-57″ and this overestimate is mitigated to some extent that the Earth has an atmosphere which boosts the angular radius of the shadow cone by about 1′ and the remainder of the overestimate makes amends for the smaller value of P taken.

Verse 7. To convert spatial measures into angular measure. The diameters of the Sun, the Moon, and Rāhu in Yōjanas multiplied by R = 3438′, and divided respectively by K_s, K_m and K_m give their angular measures.

Comm. From fig. 69, $\frac{s/2}{K_s} = \sin \hat{E} = \hat{E}$ expressed in radians $= \frac{E'}{3438}$ $\therefore$ $E' = \frac{3438 \times s}{2\,K_s}$ which means that the angular radius of the Sun is got by multiplying $s/2$ ie. the spherical radius of the Sun by R = 3438 and dividing by K_s as mentioned. Similar is the case with respect to the other two.

Note. The word Kalākaraṇa is used to signify 'To convert into angular measure.

Verse 8. An alternative method of obtaining the angular radii.

The daily motion of the Sun increased by one-tenth of its value and halved, gives the angular diameter of the Sun. The Moon's daily motion multiplied by 3 and divided by 71, gives the angular diameter of the Moon. Or the daily motion of the Moon being decreased by 715 and divided by 25 and the result being added to 29 gives the angular diameter of the Moon.

Comm. This method gives in an easy way the true angular radii. The formulae given are $s' = \frac{1}{2}s_1\,(1+\frac{1}{10})$; and $m' = \frac{3\,m_1}{74} = \frac{m_1 - 715}{25} + 29$. This may be elucidated as follows. The argument used is "If the spherical diameter of 6522 Yojanas corresponds to a spatial daily motion of $11858\frac{3}{4}$ Yojanas, what angular diameter corresponds to the angular daily motion s_1?". The proportionality is clear and the result is

$\frac{s_1 \times 6522}{11858\frac{3}{4}} = \frac{26088}{47435}$. Converting $\frac{26088}{47435}$, into a continued fraction, it is

$\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{94}$. The penultimate convergent is $11/20 = \frac{1}{2}\,(1+\frac{1}{10})$. The formula follows.

Similar calculation gives m'.

Note. The advantage of these formulae is that they are not only easy but also adopting the true daily motion we have the true angular radii. This procedure was adopted by Bhāskara from Brahmagupta. The latter, however, prescribes a nearer convergent namely 10/247 but actually $\frac{17}{420}$ is the nearest convergent.

The next formula namely $m' = \frac{m_1 - 715}{29} + 29$ is approximate. This may be elucidated as follows. Let the daily motion be 715 ; then as per the previous formula the angular diameter should be $\frac{3}{74} \times 715 = \frac{2145}{74} = 28\frac{73}{74} = 29'$ very approximately. The mean daily motion is 790 which corresponds to 32′ of angular diameter. Taking advantage of this arithmetical correlation namely that the excess of 3′ over 29′ corresponds to 75′ of daily motion. Bhāskara gives the formula $m' = \frac{m_1 - 715}{25} + 29$. This formula correctly holds good when $m_1 = 740$, for, equating $\frac{3x}{74} = \frac{x - 715}{25} + 29 = \frac{x + 10}{25}$, x will be equal to 740. For other values between 715 and above it holds very approximately. Thus, when $m_1 = 715$, $m' = 28\frac{73}{74}$ ie. 29 when $m_1 = 740$, $m' = 30$, when $m_1 = 765$, $m' = 31\frac{1}{75}$ (error $\frac{1}{75}$) when $m_1 = 790$, $m' = 32\frac{1}{37}$ (error $\frac{1}{37}$) and so on.

Verse 9. An alternative method of finding the angular diameter of the shadow cone.

$2\rho = 2/15\, m_1 - 5/12\, s$, where m_1 and s_1 are the daily motions of the Moon and the Sun respectively.

Comm. In the previous verse, we had formulae to compute the diameters of the discs of the Sun and Moon, knowing their daily motions. Since in practice we have these daily motions computed for every day, so the computation based upon those daily motions conduces to ease in the matter of calculation. Now in this verse, the radius of the shadow cone is also calculated in terms of the daily motions of the Sun and the Moon, which is more an ingenious device adopted in practice. The elucidation of the formula depends on the following technique as conceived by the Hindu astronomers. In as much as the Sun's sphere is far bigger than that of the Earth, the shadow of the Earth assumes the form of a cone. From a knowledge of the decrease in the diameter, as we proceed from the

Sun to the Earth and a knowledge of the Sun's distance, we can compute similar decrease as we proceed from the Earth to the lunar orbit knowing the distance of the Moon. Such a decrease measured in Yojanas is termed by Bhāskara as 'अपचययोजनानि' ie. Yojanas of decrease in diameter. It was this concept that led to the formulation of $2a$ in Yojanas in the form $2a = e - \frac{(s-e)\ K_m}{K_s}$ of verse 6 and is indeed based upon the similarity of triangles as proved by us in that context. Dividing the above equation by $2\ K_m$, we have

$$a/K_m = \frac{e}{2\ K_m} - \frac{1}{2}\frac{(s-e)}{K_s} \quad \text{I}$$

Dividing a, e and $(s-e)$ thus by the distances K_m and K_s is termed 'Kalā-Karaṇa' ie. converting spatial distance into angular measure. Thus dividing a by K_m is converting the radius of the shadow cone into angular measure at the lunar orbit; dividing $\frac{1}{2}e$ by K_m is estimating the angular measure of the earth's radius as seen from the Moon's distance or what is the same the horizontal parallax of the Moon, whereas dividing $\frac{1}{2}s$ by K_s is getting the angular radius of the Sun's disc as seen from the Earth and dividing $\frac{1}{2}e$ by K_s is getting the angular radius of the Earth's disc as seen from the Sun or what is the same the horizontal parallax of the Sun.

Equation I which gives a/K_m the angular radius of the shadow cone, may also be interpreted as follows. $\frac{e}{2K_m}$ = Horizontal parallax of the Moon as mentioned above which is equal to the angle (fig. 66) $\widehat{ECB}$, for, $\frac{1}{2}e$ = EB; K_m = EC so that $e/2K_m = EB/EC = \sin \widehat{ECB} = \widehat{ECB}$ expressed in radian measure. Also

$$\frac{s-e}{} = \frac{SA-GA}{SE} = \frac{SG}{SE} = \sin \widehat{SEG} = \widehat{SEG} = \widehat{EVB}$$

(alternate angle) = Semivertical angle of the shadow cone (say θ). Now $\widehat{ECB} - \widehat{EVB} = \widehat{CED}$ = angular measure of CD ie. the angular radius of the shadow cone (expressed in radian measure).

Converting $\frac{1}{2}e/K_m$ into angular measure, the proportion used by Bhāskara is "If by the daily spatial motion of $11859\frac{3}{4}$ Yojanas of the Moon, we have its daily motion in arc, what shall we have for $e = 1581$ Yojanas?" The result is $1581/11859\frac{3}{4}\, m_1$. Converting the coefficient into a continued fraction we have $\frac{1}{7+}\frac{1}{1+}\frac{1}{1+}\frac{1}{175}$......... The penultimate convergent is $\frac{2}{15}$. Hence $e/K_m = \frac{2}{15} m_1$.

Converting $\frac{1}{2}\left(\frac{s-e}{Ks}\right)$ into arc, the proportion used is "If by the daily spatial motion of $11859\frac{3}{4}$ Yojanas, we have the daily motion of s_1, what shall we have for $s-e = 6522-1581 = 4941$ Yojanas?" The result is $\frac{4941}{11859\frac{3}{4}} s_1$. Converting the coefficient into a continued fraction, we have $\frac{1}{2+}\frac{1}{2+}\frac{1}{2+}\frac{1}{146}$. The penultimate convergent is $\frac{5}{12}$. Hence, the result is $\frac{5}{12} s_1$.

Note (1) It might be asked whether the Hindu astronomers used the theory of continued fractions. The answer is, they did though they did not write the continued form in the form we do now. They arranged the successive quotients in a vertical line and called the column as a 'Valli' or 'creeper'. One may refer to the chapter in Bhāskara's Bijagamita on 'Kuttaka' in this context.

Note (2) The formula derived above to obtain tne Rāhu-Bimba or diameter of the shadow-cone at the lunar orbit, is one which could be conveniently used in practice, for, as mentioned before, the daily motions of the Moon and the Sun are ready computed for every day. Also the advantage in using this formula is that besides the fact that we need to deal only with small quantities instead of the big numbers of Yojanas, the true value for the day of eclipse is got by using the true daily motions. Using the mean daily motions, however, we have for the mean diameter, $\frac{2}{15} m_1 - \frac{5}{12} s_1 = \frac{2}{15} \times 790'\text{-}35'' - \frac{5}{12} \times 59'\text{-}8''$ $= 105\frac{2}{5} - 25\frac{2}{3} = 81$ very approximately.

Note (3) In obtaining a convergent for $\frac{s-e}{Ks}$, since the values of s and Ks are parameters s/Ks is got alright, where $\frac{e}{Ks}$ is more exaggerated as the value of the hori-zontal solar parallax. By this term the result is increased to an extent of 3′ out of which 1′ is mitigated by the fact that we have to take the earth's atmosphere also into consideration, for, that will increase the radius of the shadow to $\frac{1}{50}$th of its value.

Note (4) The last line of verse (9) is "The earth's shadow eclipses the Moon, and the Moon eclipses the Sun". This statement is deliberately made by Bhāskara to *remove the misconception in the minds of lay men* who wrongly believe the usage of the words राहुग्रस्त and केतुग्रस्त used in the calendars even today and the mythological puranic story associated with Rāhu and Kētu depicting that a serpent devours the Sun and the Moon. In fact the shadow of the earth which eclipses the Moon and the shadow of the Moon which hovers on the earth at the time of a solar eclipse do resemble the tail of a serpent.

Verse 10. To obtain the latitude of the Moon.

Vikṣepa or S'ara as it is also called ie. the latitude β of the Moon is obtained by the formula

$= \frac{\text{H}\sin\lambda \times 270}{\text{R}}$ and it will have the same direction as the Moon with respect to the ecliptic where λ is the longitude of the Moon with respect to the nearer node and 270′ or $4\frac{1}{2}^{\circ}$ is taken to be the inclination of the lunar orbit to the ecliptic or what is the same, the maximum latitude of the Moon.

Comm. The formula is evident and similar to that for calculating the declination of the Sun. Thus

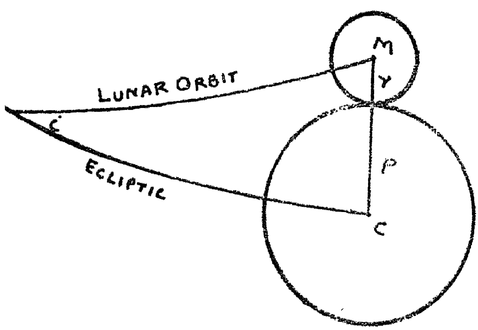
$H \sin \beta \quad \frac{H \sin \lambda \times H}{R} \quad \frac{1}{2}^{\circ}$ But since β is small and also $4\frac{1}{2}^{\circ}$, we can take $\beta = \frac{H \sin \lambda \times 270'}{R}$

The argument used by Bhāskara, however, is 'If by a H sin λ equal to R, we have the maximum latitude of 270′, what shall we have for H sin λ?' The result is as given.

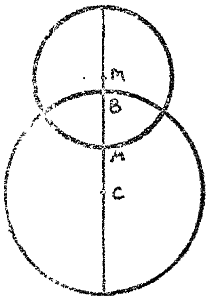
Note. The modern value for i the inclination of the lunar orbit to the ecliptic is given to vary between 4°-58′

Verse **11.** The definition of the magnitude of a lunar eclipse.

Sthagita or the magnitude of an eclipse is defined as $P+r+\beta$ where P and r are respectively the radii of the eclipsing and eclipsed bodies and β is the latitude of the Moon. If the Sthagita is greater than $2r$, then the eclipse is total.



Comm. In figure 71, the eclipsing body is just contacting the eclipsed body. Taking the case of a lunar eclipse, the latitude then of the Moon is evidently $P+r$ ie. $\beta=P+r$ holds good at the moment of first contact. (C) is the cross-section of the shadow-cone at the lunar orbit and (M) is the Moon.



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(fig. 72), $CB+AM-CM=CB+AB+BM-CM=AB+(CB+BM)-CM=AB+CM-CM=AB$

$\therefore P+r-\beta=AB=$ Sthagita. Thus Sthagita gives the portion of the diameter of the eclipsed body which is shadowed. The eclipsed body is termed the Chādya, the eclipsing body as the Chādaka and $P+r$ as the Manaikya-ardha ie. half the sum of the diameters of the eclipsing and eclipsed bodies.

When the Sthagita exceeds the diameter of the eclipsed body the eclipse is evidently total ie., when

Verse 12. Duration of the eclipse and duration of its totality.

Sthiti-Khanda Duration of the eclipse

Marda-Khanda $= \dfrac{\sqrt{(P-r)^2-\beta^2} \times 60}{m_1-s_1} = \frac{1}{2}$ Duration of totality

where P is the radius of the shadow-cone, r the radius of the Moon's disc, β its latitude taken to be constant during the eclipse, m_1 and s_1 the daily motions of the Moon and Sun respectively.

Comm. (1) The time between the moment of first contact and the middle of the eclipse or the moment of opposition or conjunction as the case may be is called *Spars'a-Sthiti-Khanda.*

(2) The time between the middle of the eclipse and the moment of last contact is called the *Mokṣa-Sthiti-Khanda.*

(3) The time between the commencement of total eclipse and the middle of the eclipse is called the *Sammīlana-Marda-Khanda.*

(4) The time between the middle of the eclipse and the end of total eclipse is called *Unmīlana-Marda-Khanda.*

The suffix Khanda meaning 'half' is generally omitted while refering to these phases. In the above verse we are given formulae for Sthiti-Khanda and Marda-Khanda only without specifying whether they pertain to Spars'a or Mokṣa. Though the same formulae serve for both the Spars'a phase as well as the Mokṣa phase under the supposition that β does not vary, it will be noted that the Spars'a-Sthiti-Khanda will not be equal to Moksha-Sthiti-Khanda and that the Spars'a-Manda-Khanda will not be equal to the Moksha-Marda Khanda in as much as β changes from moment to moment.

In fig. 73, let C_1 be the position of the eclipsing body at the moment of first contact and C_2 its position at the moment of last contact. In the figure is shown only one position of the Moon's disc signifying that we may consider the motion of the eclipsing body keeping the eclipsed body fixed (or what is the same relative to the position of the eclipsed body). It is evident from the fig. that $C_1M = C_2M = P+r$ so that $C_1M\,C_2$ is an Isosceles triangle. Let MN be the $\perp^{ar}$ dropped from M on $C_1\,C_2$. $C_1\,C_2$ is the ecliptic because the centre of the shadow will be moving along the ecliptic, for, in fig. 66, SE the ecliptic passes through D the centre of the cross-section of the shadow-cone, as well as through the vertex V of the shadow-cone.

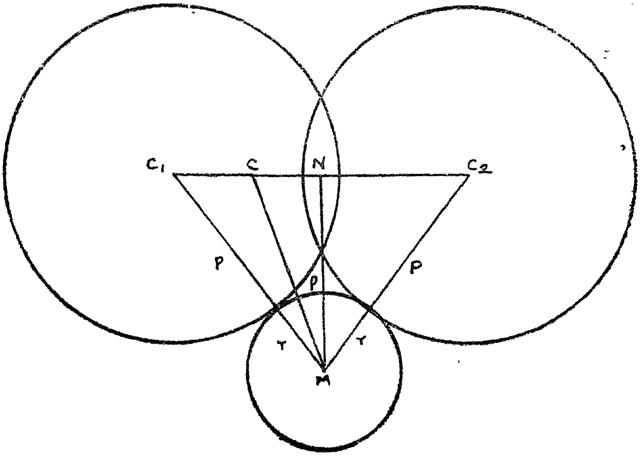


Fig. 73

MN is therefore the latitude of the Moon. Since the latitude is not the same at the moment of first contact and that of the last contact, the figure drawn does not represent the true figure but only a figure drawn on the supposition that β remains the same and C_1 Moves relative to M. From the figure $C_1N^2 = (P+r)^2 - \beta^2 =$

The Sthiti-Khanda defined in this verse is the time taken by C_1 to reach the position N ie. the position at the moment of opposition, and again from the position N to the position C_2. The velocity of C_1 relative to M is no other than the excess of the velocity of, the Moon over that of the Sun. (The velocity of the Earth is the relative velocity of the Sun with respect to the Earth and this is equal to the velocity of the shadow moving along the ecliptic). So, the time taken by C_1 to reach the position of N relative to the Moon is equal to $\frac{60 \times \sqrt{P \pm r^2}}{m_r}$

Similarly the time taken by the centre of the shadow from N to C_2 ie. from the point of opposition to the moment of last contact has also the same formula where in each case β is the latitude at the moment of opposition. The path taken by the centre of the shadow is called 'ग्राहकमार्ग' ie. the path of the eclipsing body. The actual case when both C and M are both moving and when β is considered as a non-changing quantity is shown in fig. 74. In this

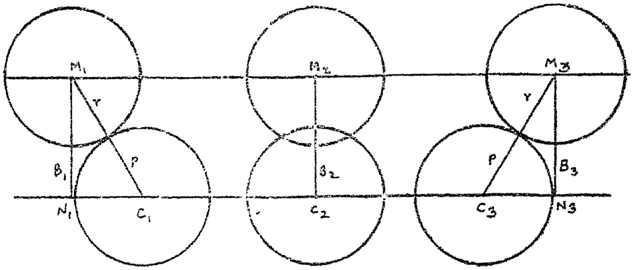


Fig. 74

case, three positions are shown, (1) that at the first contact (2) that at opposition and (3) that at last contact, where C_1, M_1, C_2, M_2 and C_3, M_3 give the positions of the centre of the shadow and that of the Moon's disc respectively, both the centres being shown as moving. Since the Moon moves faster than C and as such overtakes C, the path of M from M_1 M_3 which synchronizes with the path of C from C_1 to C_3, is shown to be longer. But, one may wonder, how C_1 N_1 and C_3 N_3 represent the Spars'a-Sthiti-Khanda and Mokṣa-Sthiti-Khanda respectively. The distance overtaken by M with respect to C from the point of first contact to the point ef opposition is $M_1 M_2 - C_1 C_2 = C_1 N_1$. Hence we compute $C_1 N_1$ by the formula $C_1 N_1^2 = (P+r)^2 - \beta^2$. Similarly from the point of opposition to the point of last contact M overtakes C by the distance $M_2 M_3 - C_2 C_3 = C_3 N_3 =$

Fig. 75 shows the situation when β changes as is the actuality. When the opposition takes place after the Moon crosses the node, then $\beta_3 > \beta_2 > \beta_1$, whereas if

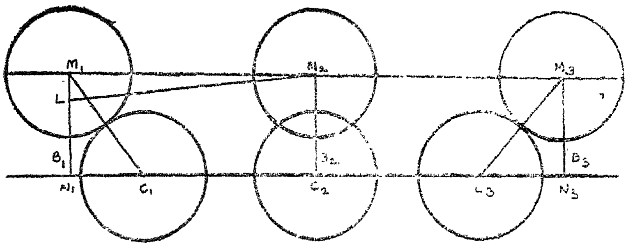


Fig. 75

the opposition precedes the Moon's position at the node $\beta_3 < \beta_2 < \beta_1$. Also, when β changes, $M_1 M_2$ does not exceed $C_1 C_2$ exactly by $C_1 N_1$. So, on both the counts, the formulae, given in verse 12 are approximate. What is done in practice is that β is computed for the moment of opposition and estimating the Spars'a-Sthiti-Khanda by the formula given above, and subtracting it from the time of opposition the moment of first contact is got. Then β is computed for that time and again the formula is applied to get the *Spars'a*-Sthiti-Khanda. Repeating the process, we rectify the Spars'a-Sthiti-Khanda. Even then, we do not have the actual value of the Spars'a-Sthiti-Khanda, because $M_1 M_2$ does not exceed $C_1 C_2$ exactly by $C_1 N_1$.

A more correct procedure would be to compute the time between the moment of first contact and the moment of opposition and by that time, to compute the length of $M_1 M_2$ and take $\frac{\quad}{2}$ in the place of m' and use the formula of verse 12. This nicety, however, need not be attended to with respect to the duration of totality, for, it does not make much difference.

Another way of obtaining a better value for T, the Spars'a-Sthiti-Khanda is to take average values for β_1 and β_2, m_1 and m_2, s_1 and s_2 where m_1 and m_2 are the values of the Moon's daily motion and s_1 and s_2 are those of

Sun's at the point of first contact and the moment of conjunction respectively.

We may also use calculus to obtain δT, the variation in time for a variation of $\delta\beta$ in β and a variation of δm_1 in m_1 ignoring the small variation in s_1, as follows.

$$T^2 = \frac{(P+r)^2-\beta^2}{m_1-s_1}$$

$$\therefore \quad 2T\,\delta T = \frac{(m_1-s_1)\times -2\beta\,\delta\beta - \overline{P+r^2-\beta^2}\,\delta m_1}{(m_1-s_1)^2}$$

$$\therefore \quad \delta T = -\frac{\beta\delta\beta}{T\,(m_1-s_1)} - \frac{\delta m_1\,(\overline{P+r}^2-\beta^2)}{T\,(m_1-s_1)^2}$$

The first term on the Right hand side gives the variation for $\delta\beta$ and the second for δm_1.

Fig. 76 shows the case of totality.

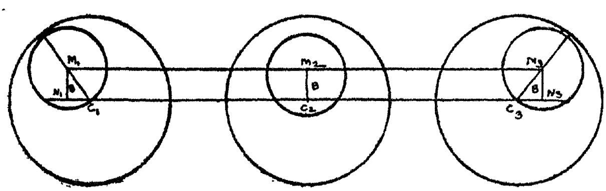


Fig. 76

$M_1 M_2 = N_1 C_1 + C_1 C_2$ $\therefore$ The Moon has to overtake C from the moment of the beginning of totality to the moment of opposition by the distance $C_1 N_1$ with a relative velocity of $m_1 - s_1$. Hence the time of Sammīlana-Marda-Khanda is equal to

$$\frac{\sqrt{C_1M_1^2-\beta^2}\times 60}{m_1-s_1} = \frac{\sqrt{(R-r)^2-\beta^2}\,6\times 0}{m_1-s_1}$$

as given, taking β to be constant. Similarly the Unmīlana-Marda-Khanda from the position ($M_2\,C_2$) to the position ($M_3\,C_3$) will also be the same, taking β to be constant.

Rectification of this time, when β is considered as varying will proceed on the same lines as before.

Verse 13. Rectification of the times of Spars'a-Sthiti-Khanda and Mōksha-Sthiti-Khanda.

From the position of the Moon and that of the Node obtained for the moment of opposition, have to be computed their positions for the moment of first contact and those for the moment of last contact. For this, $\frac{T \times v}{60}$ is to be subtracted and added respectively to the positions at the moment of opposition of the Moon and Node, where T is the time of the Spars'a-Sthiti-Khanda, and v the daily motion (of the Moon or the Node as the case may be). From these positions β has to be computed for the moment of first contact and that of last contact, and from this β the time of Spars'a-Sthiti-Khanda and Mōksha-Sthiti-Khanda have to be rectified by the method of successive approximation.

Comm. From fig. 75, $C_1 N_1$ and $C_2 N_2$ are the distances gained by the Moon over the centre of the shadow so that to get their correct values β_1 and β_2 are to be used and not β. Hence β_1 and β_2 are to be computed using T the time of Spars'a-Sthiti-Khanda and that of the Mōksha-Sthiti-Khanda which are taken to be equal in the first instance. Since T is the time taken as a first approximation, β_1, β_2 are also approximate in the first instance. From these β_1, β_2 T is to be rectified and in this rectification, we have T_1 and T_2 differing, as the times of Spars'a-Sthiti-Khanda and Mōksha-Sthiti Khanda. From these rectified times again β_1, β_2 are further to be rectified and from them again T_1, T_2 are to be further rectified. This procedure is to be continued till constant values are obtained for T_1 and T_2.

Note. T_2 will be less than or greater than T_1 according as $\beta_2 \gtrless \beta_1$.

Verse 14. Rectification of the Sammīlana-Marda-Khanda and Unmīlana-Marda-Khanda.

Proceeding on the same lines as above and obtaining β_3 and β_4 the rectified latitudes of the Moon for the moments of the commencement and end of totality of the eclipse, the Sammīlana-Marda-Khanda and Unmīlana-Marda-Khanda, T_3 and T_4 are to be rectified.

Note. We have the formula $\sin\beta = \sin\lambda \sin i$ so that by differentiating. we have $\cos\beta\delta\beta = \cos\lambda\delta\lambda \times \sin i$

$$\therefore \quad \delta\beta = \frac{\sin i \cos\lambda\, \Delta\lambda}{\cos\beta}$$

This formula gives in one stroke the rectified latitudes of the Moon at the respective moments from which the respective rectified times could be got.

Verse 15 *and the first half of verse* 16. The definition of Bhuja and the method of finding it at an intermediate point of time.

The word 'Iṣta' is used to connote 'At any given time'. The word 'Spārśika-Iṣta' means 'At a given time after the moment of first contact'; similarly the word 'Maukṣika Iṣta' means 'At a given time before the moment of last contact'. $(T-t)$ (m_1-s_1) where (m_1-s_1) is in degrees (m_1 and s_1 of course being expressed in minutes); T stands for the Sthiti-Khanda (Spārśika or Maukṣika) and t stands for the Iṣta (Spārśika or Maukṣika) gives the Bhuja. Similarly with respect to obtaining the Marda-Bhuja. (The former is called Sthiti-Bhuja).

Comm. In fig. 77, let C and M be the centres of the Rāhu (cross-section of the shadow-cone at the lunar orbit)

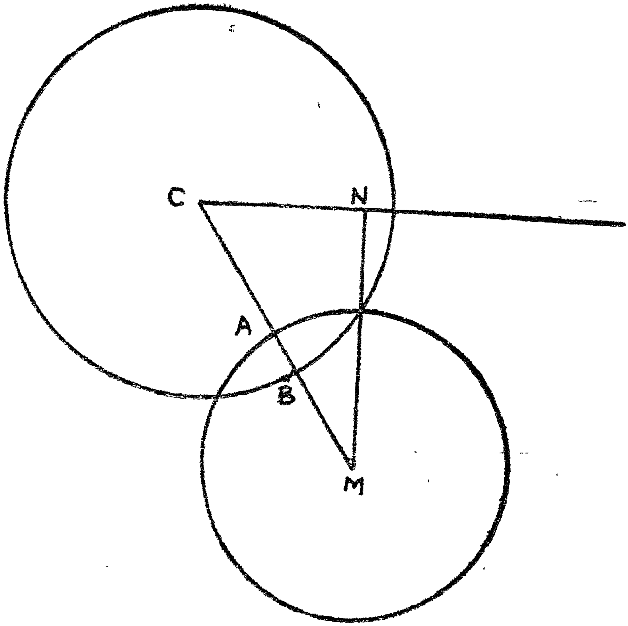


Fig. 77

and the Moon respectively; let MN be the perpendicular from M on the Grāhaka-mārga or the path of the eclipsing body (ie. the ecliptic). Then CN is called the Bhuja at the time.

At the moment of first contact, the value of CN is $\;^2-\beta^2$ where β is the latitude of the Moon at that moment. At any subsequent moment, from fig. 77, CN is equal to $\sqrt{(P+r-AB)^2-\beta^2}$ where β is the latitude at the subsequent moment and AB the portion of the radius of the eclipsed body shaded. Hence we could obtain the Bhuja at any subsequent moment, by computing the latitude at that moment and the value of AB. But AB could be computed only by knowledge of CN and β. Hence the necessity for knowing the value of CN at any subsequent moment arises. β, of course, could be computed, knowing the hourly variation of β, which in its turn could be known, by a knowledge of the hourly variation in λ, the longitude of the Sapātachandra.

The magnitude of CN is calculated by the rule of three "If by the Spars'a-Sthiti-Khanda we have initially the initial value of CN, what shall we have for (T $-t$)?" The result is $\frac{(T-t) \times CN}{T}$ where CN is the initial value of CN and T the Spars'a-Sthiti-Khanda. Substituting the values of CN and T from verse 12, where

$$CN = \sqrt{(R+r)^2-\beta^2} \text{ and } T = \frac{\sqrt{(R+r)^2-\beta^2} \times 60}{m_1-s_1}$$

we have the required Bhuja as

$$\frac{(T-t)\,(\sqrt{R+r^2}-\beta^2)}{60\,(\sqrt{R+r^2})-\beta^2)} \times (m_1-s_1) = \frac{(T-t)\,\{m_1-s_1\}}{60} \text{ minutes}$$

$= (T-t)\,(m_1-s_1)$ degrees as given.

Similarly we could find the Bhuja with respect to 'totality' ie. the 'Marda-Bhuja' as it is called.

Note. One might mistake M_1 M_2 of fig. 74 (M_1 pertaining to a subsequent moment) to be the Bhuja defined above, which is the join of the centre of the eclipsing body and the foot of the latitude at the middle of the eclipse. That is why Bhāskara uses the word 'Madhya-S'arāgra-Chihna' in the commentary, meaning thereby not the foot of the actual latitude at the middle of the eclipse but only the point N of fig. 77 which 'signifies' it.

Second half of verse 16 *and first half of verse* 17.

Taking the latitude of the Moon at a given time as Kōti, and Bhuja as the Bhuja of the moment defined above, we have the Karṇa of the moment as $\sqrt{Bhuja^2+\beta^2}$; $R+r-$Karṇa gives the Grāsa at the moment.

Comm. The word 'Grāsa' at the moment stands for AB of fig. 77, Karṇa for CM, where CN is the Bhuja and MN is the Kōti. The 'Grāsa' at the moment of opposition has the special name Sthagita.

Second half of verse 17 *and verse* 18.

To obtain the time after the moment of first contact, knowing 'Grāsa' at the moment.

$$T - \frac{\sqrt{(P+r-\text{Grāsa})^2-\beta^2}}{m_1-s_1} = t;$$ this 't' is to be rectified by obtaining the β of the moment and again finding t and repeating the process till an invariable magnitude is got.

Comm. This is the converse of finding the Grāsa given the time. The method of rectification is also evident. In the above equation considering β and t as variables, and differenting,

$$\delta t = \frac{1\times-\beta\delta\beta}{2\,(m_1-s_1)\sqrt{(P+r-g)^2-\beta^2}} = \frac{-\beta\delta\beta}{(T-t)\,(m_1-s_1)^2}$$

Knowing $\delta\beta$, δt could be got without taking recourse to the method of successive approximations.

Verse 19. Certain definitions.

The 'Middle of the eclipse' (or strictly speaking the moment when the portion eclipsed is a maximum) occurs at the moment of opposition. Spars'a or Pragraha is at the moment of first contact and Mokṣa is at the moment of last contact, separated from the moment of the middle of the eclipse by times equal to Spars'a-Sthiti-Khanda and Mokṣa-Sthiti-Khanda respectively before and after. Similarly Sammīlana and Unmīlana or the moment of the commencement of totality and the end thereof occur before and after the moment of 'the middle of the eclipse' by times equal to Sammīlana-Marda-Khanda and Unmīlana-Marda-Khanda respectively.

Comm. Clear.

Verse 20. To get what is called the Valana.

The hour-angle of the eclipsed body expressed in nādis, multiplied by 90 and divided by half the duration of night (if it be lunar eclipse) or half the duration of day (if it be solar) as the case may be will give the degrees of an angle, whose H sine being multiplied by the H sine of the latitude and divided by (H cos δ, (where δ is the declination of the eclipsed body), gives the H sine of what is called Ākṣavalana which is north when the hour angle is east, and south otherwise.

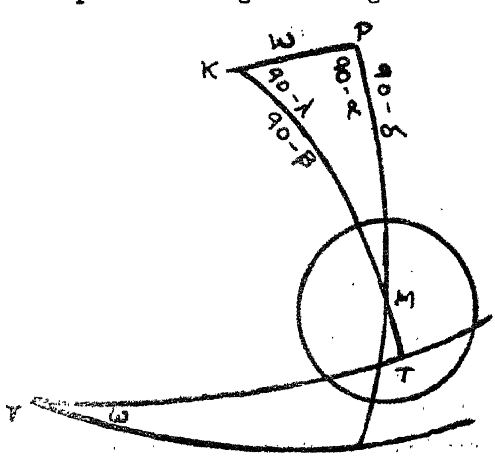
Comm. This subject of Valana requires a detailed treatment as is given in the Golādhyāya by Bhāskara. Here only a practical formula is given to proceed with the computation. For an understanding of this formula we have to necessarily draw upon the treatment in Golādhyāya.

The word 'Valana' means 'deflection'. The problem posed is at what point of the disc of the eclipsed body does the eclipse begin and at what points it ends. Since an observer sees the disc of the eclipsed body on the background of the spherical surface of the sky, the specification of the point of first contact must neeessarily be made with respect to east, west, north and south. These directions could be specified with respect great circles drawn secondary to the prime-vertical. But the Earth's shadow moves along the ecliptic and the Moon is also very nearly moving on the ecliptic at the moment of an eclipse. Thus 'Valana' should give the angle between the ecliptic and the prime-vertical; rather it should be described by two diameters of the Moon's disc, one a secondary to the prime-vertical and one a secondary to the ecliptic. In other words we have to get the angle subtended at the centre of the Moon's disc between those diameters.

This angle between the two diameters mentioned, is, for convenience divided into two parts namely $\widehat{KMP}$ and $\widehat{PMN}$, where K, P, and N are the poles of the ecliptic, celestial equator and the prime-vertical and M is the centre of the Moon's disc. $\widehat{KMP}$ is called Āyana Valana, so called because it depends upon the obliquity of the ecliptic to the Equator (अयनयोः वलनं आयनं वलनम् ie. the deflection due to the deflection of the solsticial points from the equator) whereas the angle $\widehat{PMN}$ is called Ākṣa Valana ie. deflection of a secondary to the prime-vertical namely NM with respect to a secondary to the celestial equator namely PM which is due to Ākṣa or latitude of the place.

We shall first treat the subject on modern lines and then depict Bhāskara's treatment. Let θ, ξ, η stand respectively for the Āyana, Āṣka and total Valanas respectively, where by 'total Valana' we mean $\widehat{KMN}$ which is the algebraic sum of $\widehat{KMP}$ and $\widehat{PMN}$.

From the spherical triangle KMP fig. 78.



$$\frac{\text{Sin } 90 \mp \alpha}{\text{Sin } (90-\beta)} \quad \frac{\sin \theta}{\sin \omega} = \frac{\sin (90-\lambda)}{\sin (90-\delta)}$$

$$\text{Sin } \theta = \frac{\sin \quad \cos}{\cos \delta} \text{ or } \frac{\sin \omega \cos \alpha}{\cos \beta}$$

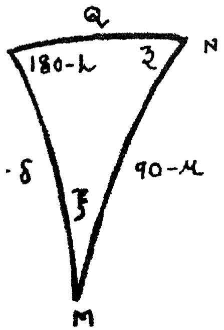


Fig. 79

Similarly from fig. 79, where P=celestial pole, N=North-point, M=centre of the Moon's disc, Q=latitude of the place, h=hour-angle of the Moon, ξ=Āksha Valana and z=Arc of the prime-vertical intercepted between the zenith and the foot of the secondary to the prime-vertical drawn through M, which arc goes by the name Sama-Vritta-Natām, sa or zenith-distance measured along the prime-vertical- μ=distance of M from the prime-vertical measured along the above secondary,

$$\frac{\text{Sin } \xi}{\text{Sin } \varphi} = \frac{\sin (180-h)}{\sin (90-\mu)} = \frac{\sin z}{\sin (90-\delta)}$$

$$\text{Sin } \xi = \frac{\sin \varphi \sin h}{\cos \mu} = \frac{\sin \varphi \sin z}{\cos \delta} \qquad \text{II}$$

In fig. 80, where (M) is the Moon's disc, AB, the diameter of the disc extending along the ecliptic (assuming the Moon's centre almost on the ecliptic, which is the case at the time of an eclipse), K, P, N respectively the pole of the ecliptic, the celestial pole and the north point and θ, ξ the Āyana and Ākṣa Valanas defined above, the eclipse starts at A, the eastern side of AB, called the Krānti-Vritta-Prāchi, AB being perpendicular to EF a diameter of the disc secondary to the ecliptic. An observer with his physical eye construes the diameter CD, which is secondary to the prime-vertical as indicating north and south. Naturally therefore, it is required to specify the

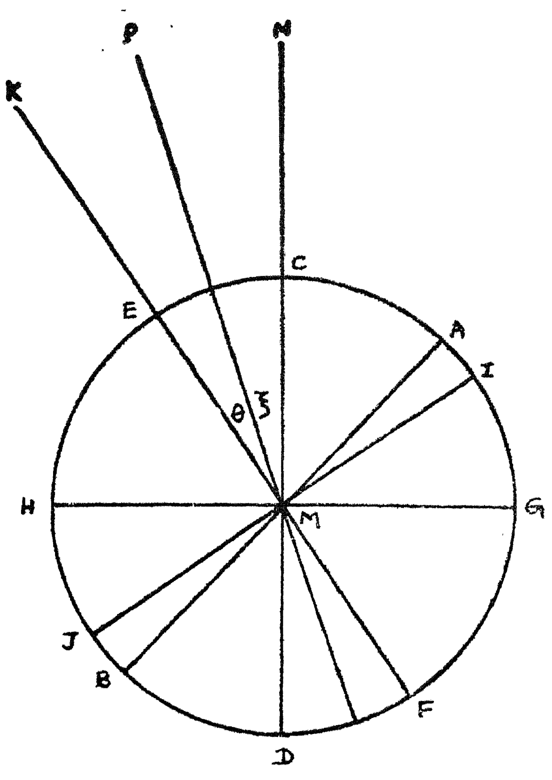


Fig. 80

location of A, the point of first contact, with respect to the diameters GH and CD, which are respectively East-West and North-South. Suppose $\overline{\theta+\xi}=45°=GMA$, then we say that the eclipse begins at the north-point of the disc and so on. For this purpose, the concept of Valana arose. We have said above that the angle $\widehat{KMN}=GMA$ is to be got, and that it is the algebraic sum of θ and ξ, meaning thereby that when K comes in between P and N, or below N, which is also likely for places of latitude less than ω, the obliquity of the ecliptic, $\widehat{KMN}$ will be equal to $\xi-\theta$ and $\theta-\xi$ respectively.

The formula given in the present verse is

$$\frac{H}{H \cos \delta}$$

$$\text{or } H \sin \xi = H \sin \left(\frac{90h}{D/2}\right) \times \frac{H \sin \phi}{H \cos \delta}$$

where h and D/2 are measured in nādis, h being the hour angle of the Moon and D/2 half-the duration of the Moon's stay above the horizon.

Evidently the formula is intended as an approximate one, for all practical purposes considered equivalent to formula II given above namely

$$\sin \xi = \frac{\sin \phi \sin z}{\cos \delta} \text{ or } H \sin \xi = \frac{\quad}{H \cos \delta}.$$

Thus in the place of z we are given $\frac{90h}{D/2}$ which means "when z=90°, D/2 is the hour angle measured in nādis, what is z when the hour angle is h?". The answer is

This formula is approximate because h and z are

not strictly in proportion though h increases or decreases along with z.

Nonetheless, the formula serves for practical purposes very approximately and the beauty lies in the concept of Sama-Vritta-Natāmśa, which means measuring hour-angle in terms of the arc of the prime-vertical instead of an arc of the celestial equator. The error, it will be noted will not be much in low latitudes.

So far with respect to the commentary on the present verse. Now we shall see how Bhāskara tackles the problem rigorously in Golādhyāya under the caption Valana Vāsanā' ie. 'concept of Valana'.

We defined above that the angle $\widehat{KMP}$ is Āyana Valana and the angle $\widehat{PMN}$ as the Ākṣa-Valana. These are respectively called Bimbīya-Āyana Valana and Bimbīya-Ākṣa-Valana being subtended at M, the centre of the Bimba ie. the disc of the Moon. If in fig. 78, T be the foot of the celestial latitude of the Moon, then the respective angles $\widehat{KTP}$ and $\widehat{PTN}$ are called the Sthānīya-Āyana-Valana and the Sthānīya-Ākṣa-Valana ie. the angles subtended at the Sthāna or the construed position of the Moon on the ecliptic.

The Āyana-Valana is zero and a minimum when M or T lies at the solstices, and a maximum equal to ω when those points lie at ♈ or ♎. Similarly the Ākṣa-Valana is a minimum equal to zero when M or T lies on the meridian and a maximum equal to ϕ when those points lie at the east or west points. In other words the Āyana-Valana increases from zero to ω as M or T moves along the ecliptic from a solstice to an equinoctial point; and the Ākṣa-Valana increases from zero to the maximum value of ϕ as M or T moves along zE or zω from z the zenith to E or ω along the prime-vertical. Hence Āyana Valana is perceived to be proportional to H sin $(90+\lambda)$ where λ is the longitude of M or T, since when $\lambda = 90$, H sin $(90 + 90) = 0$ and when $\lambda = 0$, H sin $(90+90)=0$ and when $\lambda=0$, H sin $(90+0)=R$, a maximum; similarly the Ākṣa-Valana is perceived to be proportional to H sin z where z is the Sama-Vritta-Natāms'a defined before, since, when $z=0$, M or T is at the zenith and the Ākṣa Valana is zero and when M or T is at the East or West point, $z=90°$ and H sin $z=R$, a maximum.

It is worth-hearing Bhāskara, at this juncture (Ref. verses 30–74, under the caption Valana Vāsanā pages 305–306. Ānandās'rama edition of Goladhyāya Vol. 2, Poona).

"The north and the south with respect to the Equator and Ecliptic (ie. the north-pole and south-pole) are different at the points γ and $\simeq$, being at a distance of ω from each other. Hence the Āyana Valanajyā at those points is equal to H sin 24° (ω taken to be equal to 24°). But at the solstices, the north and south will be the same (meaning thereby that the angle subtended by PK at the solstices is zero, or what is the same, the directions to the respective poles (of the Equator and the Ecliptic) at the solstitial points are the same so that the East will be the same for both the circles at those points. Thus there is no Valana at the solstices ie. P♋K=0 where ♋=cancer. In between γ and ♋, the Valana is found in proportion to H sin $(90+\lambda)$ where λ is the longitude of the point and in inverse proportion to H cos δ, where δ is the declination. Hence H sin $\theta = \dfrac{\text{H sin }(90+\lambda)}{\text{H cos }\delta} \times$ H sin ω, where θ is the Āyana Valana. Similarly at the points of intersection of the Equator and prime-vertical namely E and W, the Unmandala (the Equatorial horizon) decides the north-south direction with respect to the Equator, whereas the horizon decides the same with respect to the prime-vertical. These north-south directions with respect to those two great circles namely the Equator and the prime-vertical differ by the angle between the Unmandala and the horizon which is equal PN=ϕ, the latitude of the place. Hence at the East and West points the Ākṣa Valanajya or the H sine of Ākṣa-Valana is equal to H sin ϕ. But at the zenith, the north-south directions of the Equator and prime-vertical coincide so that there is no Ākṣa Valana at the zenith. Thus H sine of the Valana is proportional to H sin ϕ in between the points on the prime-vertical between the zenith and the East and West points. (Roughly speaking) H sin ξ = H sin ϕ H sin z where ξ=Ākṣa Valana, z=

Natāms'a (defined before) and H sin z may be taken to be roughly equal to $\frac{90h}{D/2}$ (as depicted before).

In the East the Ākṣa Valana is north, for, in fig. 80 $\widehat{GMI}$ which gives the East of the Equator with respect to the the East of the prime vertical, is north; whereas in the West $\widehat{HMJ}$ is south. (The definition of the direction of the Valana is given as a directive to add the two kinds of Valanas if they be of the same direction otherwise to take the difference; in the fig. 80, the Āyana Valana ie. $\widehat{IMA}$ is also north, so that adding $\widehat{GMI} + \widehat{IMA} = \widehat{GMA}$ is the Sphuta Valana or the actual Valana). Hence Sphuta Valana measured by GMA is had by the sum or difference of the two angles $\widehat{GMI}$ and $\widehat{IMA}$ which define respectively the Āyana and the Ākṣa Valanas.

Similarly, at the point of intersection of the Ecliptic and the prime-vertical, the Sphuta Valana is a maximum which is the sum or difference of the Valanas as the case may be. At points removed 90° on either side, from the point of intersection of the Ecliptic and the prime vertical, in as much as the north-south directions with respect to the Ecliptic and the prime-vertical coincide, the Sphuta Valana is zero.

If (as Lallācharya said) *the Valana varies* as the Hversine at those points which are *removed by 90° from the points* of intersection of the Ecliptic and the *prime-vertical, the* Sphuta Valana will not be zero (which is against common sense). Hence the Valanajya varies as Hsine and not as Hversine.

We shall look at the subject from another point of view for the sake of clarity..........Fix a circle on the sphere with the celestial pole as centre and ω as the angular radius. This circle is called Kadamba-Bhrama-Vritta or the circle in which the pole of the Ecliptic revolves round P (due to diurnal revolution of the Earth). *In that circle Hsine of an angle will be* H sin δ............Or again draw the great circle with the planet's position as the pole, called the horizon of the planet. The arc intercepted on this circle between the Ecliptic and the celestial Equator will be Āyana Valana and that intercepted between the celestial Equator and the horizon is the Ākṣa Valana; and the arc intercepted between the Ecliptic and the horizon is the Sphuta Valana.

Or again draw a circle with K as centre and radius $\omega = 24°$. This circle is called the Jina-Vritta where the word Jina means 24. Let a secondary to the Ecliptic passing through K and K′ the poles of the Ecliptic revolve with KK′ as fixed. When this revolving circle passes through Cancer (Sāyana) it will be passing through P. The angle turned through by this circle from Cancer, will be equal to the angle turned through from P. The Hsine of that angle in the Jina Vritta will be H sin δ of a longitude equal to that angle. This is the Āyana Valana and it arises at the end of Dyujyā, since the north-polar distance of the planet is $(90-\delta)$ whoes Hsine is Dyujya ie. H cos δ. The corresponding Āyana Valana in a circle of radius R is got by multiplying by R and divided by H cos δ

us clarify Bhāskara's mind. (Ref. fig. 81) Let PBD be the Jina Vritta drawn on the sphere with K, the pole of ehe Ecliptic as centre and $\omega=24°$ as radius. Let a revolving secondary to the Ecliptic coincide initially with K where is Cancer. Let it occupy subsequently the position KM where M is the centre of the Moon's disc taken to be on the Ecliptic as is the case very approximately at the moment of an eclipse. Now the Āyana Valana is the angle KMP. Let MA be the declination of to L such that MK=ML=90°. Hence

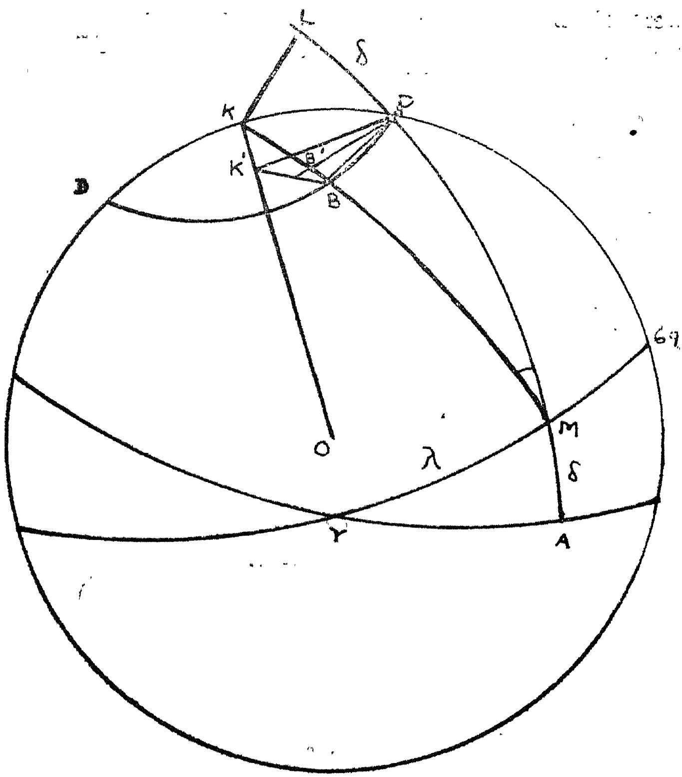


Fig. 81

$=\delta$ since PA$=90°$ and LM$=90°$. The Āyana Valana $\widehat{KMP}$ is measured by the arc ML where ML is an arc of the Grahakṣitija or the horizon of the planet M (ie. the circle with M as centre and 90° as radius drawn on the sphere or what is the same the great circle whose pole is M). PB is an arc of the small circle parallel to KL which is an arc of a great circle. Then in the Jina Vritta

sin $\quad = \sin PK \times \sin \widehat{PKB} = \sin \omega \sin (90-\lambda) = \sin \omega \cos \lambda$

$\therefore$ sin KL $= \sin \omega \cos \lambda / \cos PL = \frac{\sin}{\cos} \quad = \sin PMK.$

Here sin ω cos λ is called Sa-thribbha-graha-ja-kranti or the declination of a point whose longitude is $\overline{90+\lambda}$ where λ is the longitude of M. As we have the formula sin δ = sin ω sin λ, sine of the declination of such a point is equal to sin ω sin $(90+\lambda)$=sin ω cos λ. When δ is very small sin PMK may be taken to be sin ω cos λ or what is the same Sa-thribbha-graha-ja-krānti as is formulated by Sūrya-siddhānta.

It may be doubted how sin PB=sin PK sin $\overline{90-\lambda}$. (Ref. fig. 82). Let K′ be the centre of the circle PBD,

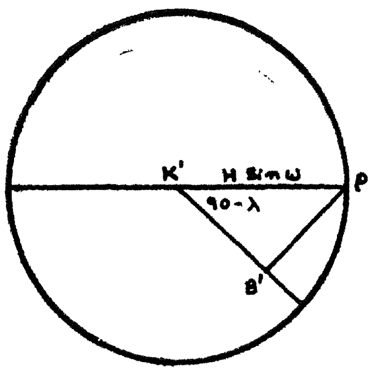


Fig. 82

K′ being in the plane of PBD. K′P and K′B are radii of this circle. Since the arc PB stands for $\overline{90-\lambda}$ PK′B = 90 $-\lambda$. Draw the H sine of arc PB, which is PB′. Now K′P=H sin ω, as PK′ is $\perp^{ar}$ drawn on OK

$\therefore$ PB′=PK′ sin PK′B

$$=\text{H sin } \omega \times \text{sin PK'B} = \frac{\text{H sin } \omega \text{ H cos } \lambda}{\text{R}}$$

$$\text{H sin KL} = \frac{\text{H sin } \omega \text{ H cos } \lambda}{\text{R}} \times \text{H cos } \delta = \frac{\text{H sin } \omega \text{ H cos } \lambda}{\ldots}$$ as

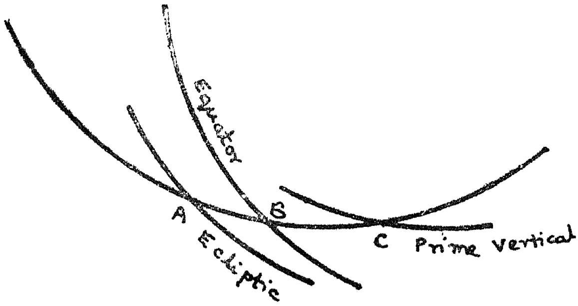


Fig. 83

Note 1. Bhāskara says PB′ (fig. 82) is Krānti-Sinjanī; so it is because $PB' = PK' \sin (90 - \lambda) = \frac{H \sin \omega \ H \cos \lambda}{R}$ = H sine of the declination of a point whose longitude is $\overline{90+\lambda}$ = Satribha-grahaja-krānti as is mentioned by Bhāskara and Sūrya-siddhānta.

Note 2. If M be the centre of the Moon's disc and ABC its horizon defined above, the arc AB intercepted between the Ecliptic and the Equator is Āyana Valana, the arc BC intercepted between the Equator and the prime-vertical is Ākṣa Valana and the arc AC intercepted between the Ecliptic and the prime-vertical is Sphuta-Valana.

Note 3. The analysis of Ākṣa Valana proceeds on similar lines, only we have P and N in the place of K and P.

Note 4. The mistake of Lallācharya alluded to by Bhāskara is as follows. The Āyana Valana, we have seen is zero at the Ayanas ie. the solstices and maximum at ♈ and ♎ ie. the equinoctial points removed by 90° from the Ayanas. Now Hversine = R−H cosine so that when $90 - \lambda = 0$ ie. $\lambda = 90°$, Hversine $(\overline{90-\lambda})$ = R−H cos $(90-\lambda)$ = R−H sin 90° = R−R = 0 and when $90-\lambda = 90°$ ie.

$\lambda=0$ Hversin $(90-\lambda)$ = R—H cos $(90-\lambda)$ = R—H sin λ = R—H sin 0° = R −0=R. Hence Lallācharya took by mistake that the Āyana Valana varies as Hvers $(90-\lambda)$ instead of H sin $(90-\lambda)$ since both Hversine and H sine of $\overline{90°-\lambda}$ are zero at the Ayanas and maximum at ♈ and ♎. The same mistake was committed by Lallācharya in the context of the Moon's phase also as criticised by Bhāskara as we shall see later. In fact, this latter criticism is not so justified as the former, as will be shown in that context.

Note 5. If instead of taking the Āyana Valana to vary as H sin $(90-\lambda)$ we happen to take according to Lallācharya that it varies as Hversine, then in places (Ref. verses 38, 39 Valana Vāsanā, Golādhyāya) removed by 90° from the points of intersection of the Ecliptic and the prime-vertical, where there should be no Sphuta-Valana, we do get that there is some Sphuta Valana there, since the value of Hversine differs from Hsine, though these two functions happen to be zero simultaneously and maximum simultaneously.

Bhāskara continues in verses 66-68 (Ibid) "I shall now depict Ākṣa Valana by means of the hour-angle. Take the sum or difference of S'anku-Agrā and S'anku-tala according as they are of the same direction or not ; compute $\overline{-B^2}$ where B is the result ; then $\dfrac{\text{H sin } \phi \text{ H sin } h}{\sqrt{R^2-B^2}}$ is equal to H sin ξ where ξ is the Ākṣa Valana

Comm. We saw before in the Tripras'nādhyāya that A=S+B where A=S'anku-Agrā, S'=S'anku-tala, and B= S'anku-bhuja = H sin μ where μ is represented in fig. 79. Hence $\sqrt{R^2-B^2}$ = H cos μ so that the above formula gives H sin $\xi = \dfrac{\text{H sin } \phi \times \text{H sin } h}{\text{H cos } \mu}$ which is the same as got by the modern formula in Equation II.

Note. A small circle parallel to the prime-vertical is called Upa-Vṛtta. Also secondaries drawn to the Ecliptic, Equator and the prime-vertical are called Kadamba-Sūtra, Dhruva-Sūtra and Sama-Sūtra. They are also called occasionally as Kadamba-prota-Vṛtta, Dhruva-prota-Vṛtta, and Sama-prota-Vṛtta.

Bhāskara proceeds to find the Āyana Valana in a very ingenious way in verses 69–74. We shall first give it a modern treatment so that we may better appreciate his genius. Let (S) be the Sun's disc. (It does not matter whether we take the Sun or the Moon). EQ is its diameter along the diurnal circle, and CL along the Ecliptic. LM is the difference of the declination of L and S. Let SL $=\triangle\lambda$ and LM $=\triangle\delta$. We have

$\sin\delta = \sin\lambda \sin\omega$;

differentating $\cos\delta \,\triangle\delta = \sin\omega \cos\lambda \triangle\lambda$

$\therefore \quad \triangle\delta = \frac{\sin\omega \cos\lambda}{\cos\delta} \times \triangle\lambda$; put $\triangle\lambda = b$, the angular radius of the disc

$$\therefore \quad \triangle\delta = \frac{b \times \sin\omega \cos\lambda}{\cos\delta} = \frac{b \,\text{H}\sin\omega \times \text{H}\cos\lambda}{\text{R} \times \text{H}\cos\delta}.$$

This gives the Valana in the disc of radius b. If that be so, what will it be on the sphere of radius R? The result is $\frac{b\,\text{H}\sin\omega \times \text{H}\cos\lambda}{\text{R}\times\text{H}\cos\delta} \times \frac{\text{R}}{b} = \frac{\text{H}\sin\omega \times \text{H}\cos\lambda}{\text{H}\cos\delta}$

as got before.

Let us hear Bhāskara, "put the disc of the Sun at the point of intersection of the Ecliptic and the diurnal circle. The Valana (LM of fig. 84) at the periphery of the disc is the difference of the declinations of L and S. To get the value of this let us first get the value. SL in terms of λ, the longitude of S. It is $\frac{b \times \text{B}}{\text{[illegible]}}$; so that LM

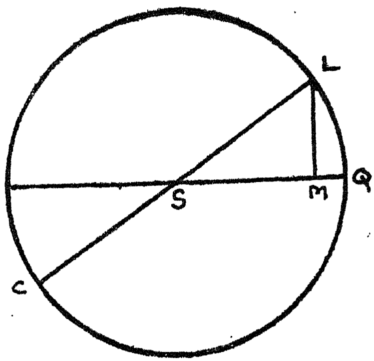


Fig. 84

will be equal to $\frac{o \times D}{225} \times \frac{I}{R} \frac{\omega}{}$ where B is the Bhogya-khanda of λ. To obtain the value of the above for a circle of radius R from a circle of radius b, we have to multiply by R/b. So, the result is

$$\frac{b \times B}{225} \times \frac{H \sin \omega}{R} \times \frac{R}{b} = \frac{B\,H \sin \omega}{225}$$

But the value of B is got as follows. 'If for $H \cos \lambda$ equal to R we have the first Bhogyakhanda equal to 225, what shall we have for $H \cos \lambda$?' The result is $\frac{225}{R}$ Substituting for B, we have

$$\frac{H \sin \omega}{225} \quad \frac{225 \times H \cos \lambda}{R} = \frac{H \sin \omega \times H \cos \lambda}{R}$$

Now, on account of declination, the Sun's disc is inclined like an umbrella. So LM of fig. 84 will take a position like L′M as shown in fig. 85 where the triangle MLL′ is similar to SMO, S being the centre of the Sun's disc, O the centre of the sphere. Hence

$$\frac{L'M}{LM} \quad R \quad = \frac{R}{H \cos \delta} \quad \frac{H \sin \omega \; H \cos \lambda}{R}$$

$H \sin \omega \times H \cos \lambda$ as got before'.

Comm. Bhāskara terms SL as the Dorjyāntara' or the variation in H sin λ, which he knows to $\frac{\text{H co}}{\text{R}}$

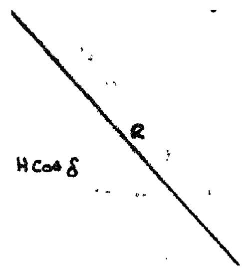


Fig. 85 Fig. 86

But proceeding from first principles, as he always does, he asks us to consider the Bhogyakhanda at λ namely B. If this be for an interval of 225′, what will be it be for 'b'? The result is $\frac{b \times \text{B}}{225}$ Then to rectify B, the proportion used is as used above. Bhāskara says many a time that the variation in Hsine is proportional to Hcosine. This concept he might have derived by looking at the Hsine table of 90 Hsines. Hence the argument advanced by him to rectify B is 'If for Hcosine equal to R (at zero-value of the argument) the initial Bhogyakhanda is 225, what will it be for an arbitary H cos λ ? The result is

$\frac{\text{H cos } \lambda}{\text{R}} \times 225$. Substituting this for B in the above, we have $\frac{b \text{ H cos } \lambda}{225} \times 225 = \frac{b \text{ H cos } \lambda}{\text{R}}$. This expression we perceive as no other than $\frac{\text{H}}{\text{R}}$ as to $\triangle$ (H sin λ), for b is to be taken as $\triangle\lambda$. $\frac{b}{\text{R}}$ is called by as Dorjyāntara meaning thereby $\triangle$ (H sin λ).

Then the next argument is 'If for H sin λ equal to R we have the declination equal to H sin ω, what shall we have for the above Dorjyāntara. The result is

$$\frac{\text{H} \sin \omega}{\text{R}} \times \frac{\text{H} \cos \lambda \times b}{\text{R}}.$$ Since this is in modern terms

$\sin \omega \cos \lambda \times b$ and $b = \triangle\lambda$, we perceive that this expression is $\triangle$ (H sin δ) where δ stands for LM of fig. 84, ie. the difference of the declinations of the points S and L of the disc of fig. 84. The next argument advanced by Bhāskara, namely that on account of declination, the disc is slanted and LM gets thereby enlarged into L′M of fig. 84 and adduces proportionality from fig. 86. But this argument seems to be faulty. In fact, the magnitude of LM is got for the diurnal circle of radius H cos δ. To get its value for radius R, the result would be

$$\frac{\text{H} \sin \omega \text{ H} \cos \lambda \times b}{\text{R}^2} \times \frac{\text{R}}{\text{H} \cos \delta} = \frac{b.\text{H} \sin \omega \text{ H} \cos \lambda}{\text{R}.\text{ H} \cos \delta}.$$

Then the argument is 'If in the disc of radius b, we have $\triangle\delta$ equal to the above what will it be for radius R? The result would be $\frac{\text{H} \sin \omega \text{ H} \cos \lambda}{\text{H} \cos}$

Note. Our argument is based on the idea that lines of the small circle namely the diurnal circle get enlarged for a circle of radius R in the porportion R: H cos δ. Bhāskara's concept of enlargement on account of slanting does not seem to be plausible because, on account of declination, the disc may occupy an overhead position when the Equator is itself inclined. Thus slanting does not arise out of declination.

The question might be asked as to how Bhāskara got the right answer by such an argument. He got the answer up his sleeves through the other methods he gave and he adduced this argument to get at that answer.

Second half of verse 21 and first half of verse 22.

$$\frac{H\cos\lambda \times H\sin\omega}{H\cos\delta} = H\sin\theta \text{ where } \theta \text{ is the Āyana}$$

The direction of this Valana is that of the hemisphere north or south in which the Moon lies.

Comm. This is the formula we have already derived. Regarding the direction of the Āyana Valana, the convention is that it is to be considered north, if the Moon be in the northern hemisphere, otherwise south. The reason is that at the time of a lunar eclipse, the Moon being in opposition, if he be north, the Sun will be south of the Equator, and fhe line BA of fig. 80 representing the Ecliptic which is roughly the join of the Sun and the Moon will be north of the line JI which is parallel to the equator. Thus the direction of the angle IMA gives the direction of the Āyana Valana.

Latter half of verse 22 and verse 23. Sphuta Valana.

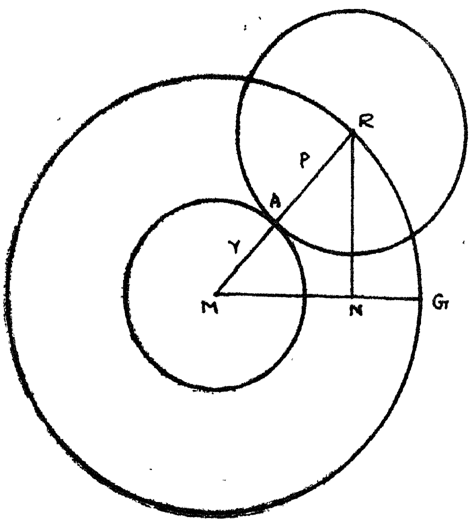
The Hsine of the sum or difference of the two Valanas according as they are of the same or opposite directions, multiplied by the sum of the angular radii of the Moon and Rāhu, and divided by the radius gives the Hsine of the Sphuta Valana. Those who said that the Valana is proportional to the Hversine, do net know spherical geometry properly.

Comm. The direction of the Ākṣa Valana, was defined in verse 20 that it is north if the hour angle is east, otherwise south. The meaning of this convention is that the diameter of the Moon's disc parallel to the Equator when the hour angle is East, is north of the diameter which is parallel to the prime-vertical. Thus combining the two conventions regarding the directions of the Valanas, it is clear that if both the Valanas are north, the line MI is north of MG, and MA is north of MI (fig. 80) so that the

Sphuta Valana is equal to the sum of the angles $G\hat{M}I$ and $I\hat{M}A$. Suppose MA is south of MI either falling within the angle GMI or south of MG, then clearly the Sphutr Valana $G\hat{M}A = G\hat{M}I - A\hat{M}I$ or $A\hat{M}I - GMI$ as the case may be, which is obtained as the difference of the two Valanas. Having obtained $G\hat{M}A$ as the Sphuta Valana, H sin GMI multiplied by $(P+r)$ and divided by R gives Hsine of the Sphuta Valana to be represented in a circle of radius equal to $\overline{P+r}$. This latter convention of representing the Sphuta Valana in a circle of radius $\overline{P+r}$ is only a convention. The expression

$$\frac{\text{H sin (Sphuta Valana)} \times \overline{P+r}}{R}$$ gives us RN of fig. 87,

where GMA is Sphuta Valana. In other words, we are to draw fig. 87 to show the point of first contact namely A in relation to MG the line parallel to the prime-vertical.



Verse 24. Conversion of liptas into what are called Angulas.

H cos z of the eclipsed body at the moment of eclipse being divided by the radius and the result being added to $2\frac{1}{2}$ gives the number of liptas per angula. The time elapsed after the rise of the body being divided by the rising hour angle (both being expressed in the same units of time) and the result being added to $2\frac{1}{2}$ also gives the same.

Comm. While a parilekha or a geometrical drawing of the eclipse is attempted at, the problem arises as to how many liptas or minutes of arc giving the measure of the disc are to be taken to be equivalent to one angula. For example, suppose the diameter of the disc is 30′. With what radius shall we draw the disc on a board or paper? In this behalf, a convention based on observations is being mentioned. The discs of the Sun and the Moon are observed to be big at rise and small when they are on the meridian. So, taking the measure of the disc to be 30′ for example, if the eclipse takes place at the rise of the disc, it is laid down to draw the disc with a radius of $15/2\frac{1}{2}$ ie. 6 angulas at the rate of $2\frac{1}{2}$ liptas per angula. {The word angula here mentioned might not be what we take it to be today in our daily parlance as one inch. The gnomon or S'anku was taken in those days to be of a length equal to 12 angulas. Bhāskara mentions in the beginning of Lilavati that 8 Yavas are together equal to one Angula, twentyfour angulas to one hasta, four hastas to one danda and 2000 dandas to one Kros'a, and 4 Kros'as to one Yojana. Also a vamsa is equal to ten hastas. This system discloses that, a Yojana equals 5 modern miles according to Bhāskara's estimate of the diameter of the Earth as compared with its modern estimate. (The method given by Bhāskara as to how the diameter of the Earth could be measured is found to be quite scientific as mentioned by us before).

33 modern inches or angulas are equal to 80 angulas of Bhāskara as per the above.

At this rate the gnomon's modern length would be

Reverting to our subject, we are asked to represent the disc of 30 liptas when the eclipse takes place at noon by $30/3\frac{1}{2}$ angulas counting at the rate of $3\frac{1}{2}$ liptas per angula.

Then the question arises as to what should be the correspondence between the liptas and angulas when the eclipse takes place in between the rise of the Moon (or Sun) and its noon. The directive is that

$$\text{one angula} = 2\tfrac{1}{2}' + \frac{\text{H} \cos z}{\text{R}} = 2\tfrac{1}{2} + \cos z \quad \text{I}$$

This means, supposing Z=zenith-distance of the body to be 60°, one angula is to be taken to be equal to $2\frac{1}{2} + \cos 60 = 2\frac{1}{2} + \frac{1}{2} = 3'$ or 3 liptas.

The reason given by Bhāskara reiterating what Sripati said in that behalf, as to why the Moon's or Sun's disc appears to be big at the moment of rising and small on the meridian, is that the disc is immersed in its own rays at noon and rendered small in appearance, whereas, most of the rays are swallowed by the earth or its atmosphere at the moment of rising, making the disc appear large and easily visible.

Note. Bhāskara gives the proof of the above formula I as follows. Since at the time of rise, we are taking $2\frac{1}{2}$ liptas of the measure of the disc to be equal to one angula and while the disc is on the meridian, $3\frac{1}{2}$ liptas are to be taken as one angula, there is an increase of one lipta for an increase of H cos z from zero at the horizon to a value equal to the Radius. So, the argument adduced is 'If for an increase of H cos z equal to the radius, there is an increase of one lipta in addition to $2\frac{1}{2}$, what should the increase be for an arbitrary H cos z'? The result is

$$\frac{H \cos z}{R} \times 1 = \cos z.$$

Note 2. As finding cos z at the time of an eclipse implies additional arithmetical calculation, and as we have already with us the data of (1) the time elapsed after rise of the celestial body (Moon or Sun) till the moment of the eclipse and (2) half the duration of the day of the body ie. the rising hour-angle converted into time at the rate of 6° per nadi, so a rough formula is given using h in the place of z. The rule of three now used, is 'If for an unnata equal to the dinārdha or rising hour-angle converted into nadis, we have an increase of 1 lipta per angula (over and above $2\frac{1}{2}$ liptas) what shall we have for an arbitrary unnata?' The result added to $2\frac{1}{2}$ liptas, gives the formula one angula $= 2\frac{1}{2} + \frac{\text{Unnata}}{\text{Dinārdha}}$. Dinārdha corresponds to H, the rising hour-angle and Unnata corresponds to $(H-h)$ where h is the hour-angle at the time.

Bhāskara uses the word 'Angula-liptas' meaning thereby the liptas that are to be taken to be equal to one angula while drawing the parilekha of the disc at the time of its eclipse.

Verse 25. Converting Valana etc. into Angulas.

The measures of the Valana (defined above) or the S'ara ie. the celestial latitude of the Moon or the Rāhu Bimba or the Bhuja (defined) are to be converted into Angulas at the rate given by the above formula. While drawing a figure of the solar eclipse, the celestial latitude of the Moon is to be drawn in its own direction whereas in a lunar eclipse, it has to be drawn in the opposite direction.

Comm. The first part is clear. Regarding the second statement, in as much as the centre of the Rāhu Bimba lies at the foot of the Moon's celestial latitude, the latter has to be drawn in the opposite direction, since in the parilekha, the centre of the Moon's disc is to be at the centre.

Verses 26 to 29. How to depict the eclipse in drawing.

Draw a circle with radius equal to that of the radius of the disc of the eclipsed body and also a circle of radius equal to $r+p$, the sum of the radii of the eclipsed and eclipsing bodies; let directions (east etc.) be marked in the figure. In the outer circle, draw the Valanajya or the Hsine of the Sphutavala with respect to the East point, Valanajyā pertaining to the moment of first contact. In the case of the Moon, the Valanajyā pertaining to the moment of first contact should be marked from the East point and that pertaining to the moment of last contact should be marked from the West point. In the case of the Sun the reverse is to be done. If the Valana is south, it should be marked in the clockwise direction, otherwise anticlockwise.

Having marked the Valanajyā in the form of a Hsine, draw the line joining the centre to the top of the Valanajyā, ie. to the point of intersection of the Hsine with the outer circle. The celestial latitude of the Moon is to be drawn from this top of the Valanajyā in the form of Hsine again. If the latitude pertains to the moment of first contact, it should be drawn from the top of the Valanajyā pertaining to that moment, and if it pertains to the moment of last contact, it should be laid off from the top of the Valanajyā pertaining to the moment of last contact. The celestial latitude pertaining to the middle of the eclipse should be drawn from the centre along the line of Valanasūtra or the line joining the centre to the top of the Valanajyā. Taking the extremities of these latitudes, circles are to be drawn with the radius of the eclipsing body to depict the eclipse at the respective moments.

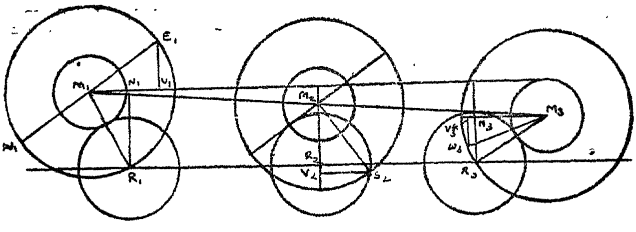


Fig. 88

Comm. Let M_1, M_2, M_3 be the positions of the centre of the Moon's disc at the moment of first contact, at the middle of the eclipse and the moment of last contact respectively. Draw a circle with radius $M_1 R_1 = r+p$ which is called the Manaikyārdha Vritta. Let $M_1 E_1$ represent the Eastern direction known as the Samamandalaprāchī. Draw $E_1 V_1$ equal to Hsine of the Valana, so that $M_1 V_1$ is the Krānti-Vritta-prāchī ie. the point of intersection of the Ecliptic with the Eastern horizon. Draw $N_1 R_1$ perpendicular to $M_1 V_1$ in the form of Hsine, which is the latitude (Vikṣepa) of the Moon at the moment of first contact. With R_1 as centre draw the Grāhaka-Vritta with p as centre; this circle represents the Rāhu-Bimba.

Similarly let M_3 be the centre of the Moon's disc at the moment of last contact. Draw a circle with M_3 as centre and $r+p$ as the radius which is the Manaikyārdha-Vritta. Let $M_3 W_3$ be the direction to the West, the Samamandalapratīchī. From W_3 draw $W_3 V_3$ the Hsine of the Valana, so that $M_3 V_3$ is now the Krāntimandala-pratīchī ie. the point of intersection of ecliptic with the western horizon. Let, the Vikṣepa $N_3 R_3$ be drawn as a Hsine of the Mānaikyārdha Vritta. With R_3 as centre and radius p, draw the Rāhu-Bimba.

Let M_2 be the centre of the Moon's disc at the middle of the eclipse. Let $M_2 S_2$ represent the South with respect to the prime-vertical. From S_2, draw the Hsine of the Valana $S_2 V_2$ so that $M_2 V_2$ is the Krānti-Vritta-Dakshinā ie. the South with respect to the Ecliptic. Now the

Vikṣepa or the celestial latitude of the Moon $M_3 R_2$ is to be drawn along this Valanasūtra M_2 V. With R_2 as centre and radius p drawn the Rāhu-Bimba.

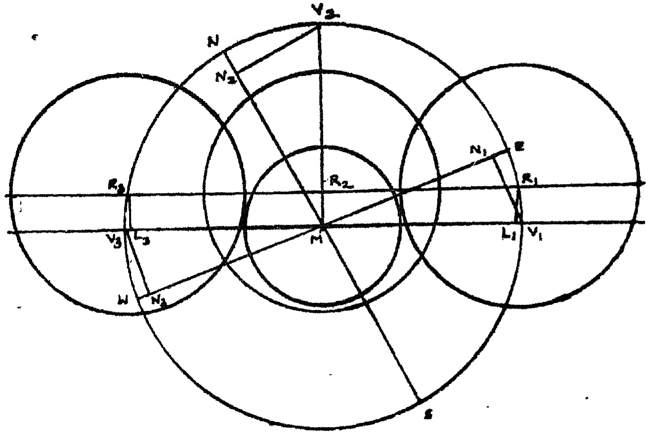


Fig. 89 Depiction of Fig. 88, keeping the Moon fixed

The slight flaw in this figure is that $ML_1 L_3$ implied as the path of the Moon is taken to be parallel to the ecliptic $R_1 R_2 R_3$ the path of the eclipsing body the Grāhakamārga, in as much as latitudes are drawn perpendicular to $ML_1 L_3$.

This figure depicts a total eclipse of the Moon. If M coincides with R_2 at the middle moment of the eclipse, then the eclipse is called central.

The duration of a central eclipse will be on the average the time that the Moon's disc takes to cross the diameter of the Rāhu-Bimba with its relative velocity. Hence the mean duration of a central eclipse is

$$\frac{\text{Average diameter of Rāhu} + \text{Average diameter of the Moon}}{\text{Relative velocity of the Moon with respect to the shadow}}$$

$$= \frac{(81+64) \times 24}{790'\text{-}35'' - 59'\text{-}8''} \text{ hrs} = \frac{145}{732} \times 24 = \frac{290}{61} =$$

4 hrs–45 minutes approximately.

Verse 30 *and first half of* 31. Geometrical depiction of the eclipse at the beginning and end of totality and also of the magnitude of the eclipse.

The Bhuja is to be laid from the centre of the Moon along its Valanasūtra or the line indicating the direction of the ecliptic; the latitude is to be drawn from the end of the Bhuja and perpendicular to the Bhuja. The hypotenuse is to be drawn from the centre of the Moon. Taking the point of intersection of the latitude (Kōti) and the hypotenuse, as centre and radius p equal to that of the eclipsing body, if circles be drawn, from these circles could be known the points where totality begins and ends as well as the magnitude of the eclipse at any given moment. Or these could be found in another way as follows.

Comm. The method given above for depicting the phases of an eclipse geometrically, could be applied for any moment during the course of the eclipse and depends upon before-hand computed Bhuja and Kōti. Refer to fig. 90. Let M be the centre of the Moon's disc. Mark Eω the East-west line drawn through M. Compute the Valana for the required moment, either for the moment when totality begins or for that when totality ends or for

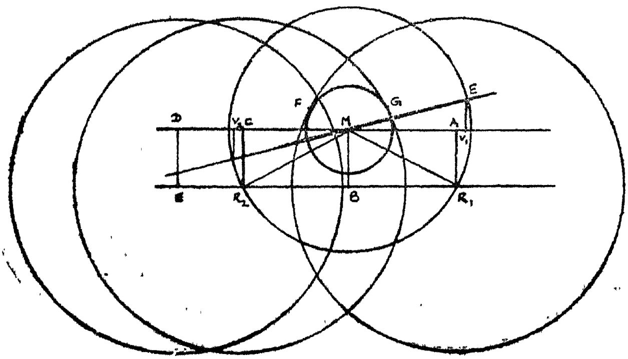


fig. 90

any arbitrary moment whatsoever. With this Valana primarily laid in the Manaikyārdha Vritta, decide the Krānti Vrittaprachī or the East-West direction of the eclipse. Thus in the figure V_1 V_2 is this direction. Then lay off the computed Bhuja along this V_1 V_2 from M, say MA for the Sammīlana moment or the moment when totality begins or MC for the Unmilana moment or the moment when totality ends or MD for an arbitrary moment. Draw AR_1 or CR_2 or DE equal to the latitude at the particular moment, perpendicular to the Valanasūtra. In the figure drawn the Valanasūtra is shown to be the same. This does not mean it will be the same throughout. It will be changing because the position of the Ecliptic changes from moment to moment. So Bhāskara uses the word ie. 'the respective Valanasūtra'. Also the latitudes will be differing from moment to moment as well as the Bhujas both of which are to be computed for any moment along with the Valana. (The method of computing the Bhuja was given in verse 15). Computing the respective Karṇas or the hypotenuses from the formula $K = \sqrt{Bhuja^2 + K\bar{o}ti^2}$, (Kōti is here the latitude) with centre M and radius equal to the Karṇas, if arcs be drawn to cut the latitudes, the points of intersection would be no other than R_1, R_2 or E. Join MR_1, MR_2 and ME. With centres R_1 and R_2 and radii equal to $p-r$, (where p is the radius of the Rāhu-Bimba, and r the radius of the Moon's disc) if circles be drawn, they just touch the Moon's disc at F and G which are the points where totality begins and ends respectively. With centre E and radius P, if a circle be drawn, that will show what amount of the disc is shadowed as well as the measure of the magnitude of the eclipse (defined in verse 11).

Note. In the above commentary and figure we have depicted MD as the Iṣta-Bhuja or the Bhuja at a given moment, taking a moment prior to the Unmīlanakāla, for showing the magnitude of the eclipse.

If a moment in between the Sammīlana and Unmīlana were taken, the then Bhuja and Koṭi could be no doubt computed, but the question of magnitude of the eclipse does not arise as the entire disc has been plunged in the shadow.

Second half of verse 31 *verse* 32 *and first half of verse* 33. Alternative method of depicting the eclipse geometrically.

Joining the upper end of the latitude of the middle moment of the eclipse to those of the first and last contacts, we have what are called the Pragrahamārga and Mokṣamārga ie. the path of the centre of the eclipsing body from the first contact to the middle moment of the eclipse and that from the middle moment to the last contact. The lengths of these paths could be computed and they could be drawn before hand. Then with the centre of the Moon as centre and radius equal to $p-r$, if a circle be drawn, it cuts the paths described above each in one point. With those points as centre and radii equal to p, if circles be drawn, they will touch the Moon's disc each in one point which are respectively the points of Sammīlana and Unmīlana.

Comm. In as much as the latitude of the Moon differs from moment to moment, the Pragrahamārga and the Mokṣamārga are separated to achieve a little more accuracy than could be got by joining the upper extremities of the initial and final latitudes. The remaining statement is evident, for, at the moments of Sammīlana and Unmīlana, the distance between the centres of the eclipsing body and the eclipsed will be $p-r$, so that the points of intersection of the Pragrahamārga and Mokṣamārga with the circle whose centre is the centre of the eclipsed body and radius $p-r$ will give the centre of the eclipsing body.

Latter half of verse 33. To know the magnitude of the eclipse at any given moment during the course of the eclipse.

Let the product of the time elapsed from the moment of first contact and the length of the path of the eclipsing body traced from the moment of the first contact to the middle of the eclipse divided by the time between the moment of first contact and the middle of the eclipse, be x. Similarly let the product of the time before the end of last contact and the path of the eclipsing body traced between the middle moment of the eclipse and the moment of last contact divided by the time between the middle moment and the moment of last contact be y. Lay off x and y units of length from the first and last points of the path of the eclipsing body along the path respectively. Then we get the points of the centre of the eclipsing body at the required moments. With these points as centre and radius p, if circles be drawn, they represent the eclipsing body. The length of the diameter of the eclipsed body shaded, gives the magnitude of the eclipse called grāsa.

Comm. Here rule of three is applied namely "If during time T_1 or T_2 a path equal to l_1 or l_2 in length is traced what length will be traced in times t_1 or t_2?", where T_1 and T_2 are the times called Sparśa-Sthiti-Khanda and Mokṣa-Sthiti-Khanda respectively, l_1 and l_2 are the times elapsed from the moment of first contact or before the moment of last contact and t_1, t_2 are the times from the beginning of the eclipse and before the end of the eclipse respectively. Then x and y give the points where the centre of the eclipsing body lies.

Verse 35. Given the magnitude of the eclipse at any time to obtain the time elapsed after the first contact.

The time taken by the centre of the eclipsing body to move through the segment of the path of the eclipsing body which lies between the position of the eclipsing body at the moment of first contact and the point of intersection with the path of the eclipsing body of the circle drawn with the centre of the eclipsed body as centre and radius equal to the difference of $p+r-g$ where g is the magnitude of the eclipse (grāsa) at the moment, or similarly the time taken by the centre of the eclipsing body to move through a similar and equal segment of the path of the eclipsing body on the other side, gives the time elapsed after the moment of first contact or the time before the moment of last contact.

Comm. This is the converse of the above problem. The method is clear being based on rule of three as above. Both the problems could be algebraically expressed as follows. Let T, t, l, g, and k, stand respectively for the Sthiti-Khanda ie. the time between the moment of first contact to the middle of the eclipse or the time between the middle moment to the moment of last contact; (2) the time elapsed after the moment of first contact or the time before the moment of last contact, as the case may be; (3) the length of the Pragrahamārga or Mokṣamārga; (4) the grāsa which is defined as $p+r-k$; (5) the Karṇa whose expression is $\sqrt{B^2+\beta^2}$, B being the Bhuja defined and β the latitude of the Moon.

Then the following working is stipulated (a) If in time T, a path of length l is traced, what will be traced in t? The result is $\frac{lt}{T}$ (b) Then $B = l - \frac{lt}{T}$ (c) $B^2 + \beta^2 = K^2$ (d) $p+r-k=g$. Thus combining all the steps

$$l^2 (T-t)^2 + \beta^2 T^2 = T^2 (p+r-g)^2 \quad \text{I}$$

given t, this equation gives g and given g it gives t.

Again the following relation holds good between T and l, $l^2 = (p+r)^2 - \beta^2$ II and

$\frac{l}{m_1 - s_1}$ = T with the nomenclature already employed which

means $T = \frac{\sqrt{(p+r)^2 - \beta^2}}{m_1 - s_1}$ III

In the above working, the fundamental elements are p, r, β, m_1 and s_1 with which the other elements could be worked out. Replacing the other elements from equation I, we have

$$\{(p+r)^2 - \beta^2\} \left\{ \frac{\sqrt{(p+r)^2 - \beta^2}}{m_1 - s_1} - t \right\}^2 +$$

$$\beta^2 \left(\frac{\overline{p+r}^2 - \beta^2}{(m_1 - s_1)^2} \right) = \frac{(p+r)^2 - \beta^2}{(m_1 - s_1)^2} (p+r-g)^2$$

ie. $\{(p+r)^2 - \beta^2\} [\sqrt{(p+r)^2 - \beta^2} - t\,(m_1 - s_1)]^2 +$

$$\beta^2 (\overline{p+r}^2 - \beta^2) = (\overline{p+r}^2 - \beta^2)(p+r-g)^2$$

ie. $\{\sqrt{\overline{p+r}^2 - \beta^2} - t\,(m_1 - s_1)\}^2 + \beta^2 = (p+r-g)^2$ IV

Putting $t=0$ in this equation, we have $(p+r)^2 = (p+r-g)^2$ ie. $g=0$ which means at the moment of first contact, the grāsa is zero. Again putting

$t = T$ ie. $t = \frac{\sqrt{(p+r)^2 - \beta^2}}{m_1 - s_1}$ ie. $t\,(m_1 - s_1) = \sqrt{\overline{p+r}^2 - \beta^2}$

we have

$\beta^2 = \overline{p+r-g}^2$ ie. $g = p+r-\beta$ which gives the grāha at the middle of the eclipse which was defined as the Sthagita. In equation IV which we may take as a fundamental equation, the two unknowns are t and g one of which being given the other could be got.

Verse 36. The colour of the eclipe.

When less than half the disc of the Moon is eclipsed, the colour will be what is called Dhumra ie. of the colour of smoke; when the disc is half eclipsed, the colour is black; when more than half is eclipsed, the colour would be a blend of black and red and when the entire disc is eclipsed, the colour will be what is called piśanga or reddish-brown.

Comm. Clear.

Verse 37. When declare the occurrence of an eclipse.

When even one-sixteenth of the diameter of the Moon's disc is shadowed, the eclipse will not be visible in as much as the shadowed portion is covered by the illuminating rays of the disc. In the case of the Sun, when even one-twelfth of the diameter is shadowed, the eclipse will not be visible for the same reason. Hence we shall not declare the occurence of an eclipse upto the shadowing of the discs to the extents stated above.

Verses 38 and 39. Examples which disclose the invalidity of construing Valana in terms of Hversine instead of Hsine.

When the Sun is in the zenith, the Ecliptic being vertical, the Valana is clearly seen to be the Agra of $(\odot+90)$ where $\odot$ is the longitude of the Sun. If you could show that the Valana will be the same on the basis of Hversine–formula, then I would accept that what Lallāchārya postulated in his work Siṣya-Dhī-Vṛddhidā is correct.

Again, in a place of latitude $90-\omega$, ω being the obliquity of the Ecliptic (ω is taken to be $24°$), when the Sun being situated in Meṣa, Vṛṣabha, Mīna or Kumbha, the Moon contacts him from the south at the moment of a solar eclipse, in as much as the Ecliptic coincides with the horizon. In this circumstance, how could the Valanā be equal to R, as made out by the Hversine–formula.

Comm. Lallāchārya gave the Valana in terms of the following verses "स्पशाद्दिकालजनितोत्क्रमशिञ्जिनीं

वर्णेन

कृ क्रमेण; ग्राह्यात् सराशोत्रितयात् भुजज्

क्रमज्या..." Verses 23, 25 Chandragrahaṇādhikāra, wherein he formulated the Valana in terms of Hversine in the place of Hsine. The reason for his slip, we have already explained. Now Bhāskara gives two glaring examples to substantiate his formula and to show up the flaw in Lallācharya's formulation.

In the first example, where the Ecliptic takes the form of a Vertical, the Sun being in the zenith, the Spaṣṭa Valana which is the angle between the Ecliptic and the prime-vertical is the same as the arc between the East point and the intersection of the Ecliptic with the horizon known as Lagna. Since the Sun is then in the zenith, the longitude of the Lagna is $(90+\odot)$ so that the said arc is the Agra of the point whose longitude is $\overline{90+\odot}$ as stated. Hence Spaṣṭa Valanajyā $= \sin A = \sin \delta / \cos \phi$ where A is the agrā (using Napier's rule from triangle PNL where L is the Lagna N the north-point and P the celestial pole). In the Hindu form, this is given by $H \sin V = H \sin A = \dfrac{R\ H \sin \delta}{H \cos \phi}$ where δ is the declination of a point of the Ecliptic whose longitude is $(90+\odot)$. But Lallāchārya's formula gives the Valanajyā as Hvers δ, δ being the declination of a point of the Ecliptic whose longitude is $(90+\lambda)$, λ being the longitude of the Eclipsed body ignoring the latitude. In other words, in the case of the lunar Eclipse when the Moon is in the zenith his Valanajyā $=$ Hvers δ (δ having the above value) the Ākṣa Valanajyā here being zero. Since $\dfrac{R\ H \sin \delta}{H \cos \phi} \nless$ Hvers δ, the mistake committed by Lallāchārya is evident even supposing $H \cos \phi = R$ when we ignore the latitude ie. take ϕ to be zero.

In the second example cited by Bhāskara the Ecliptic coincides with the horizon, the pole of the Ecliptic being in the zenith. Then in a Solar Eclipse the Moon eclipses the Sun from the south showing that H sin V = R. That H sin V = R is also evident from the fact that the Ecliptic makes 90° with the prime-vertical, having coincided with the horizon. But here according to Lallāchārya's formula, H sin ξ = Hvers 90° = R and Āyana Valanajyā is Hvers δ, where δ is the declination of a point whose longitude is 90° more than ⊙. If ⊙ = 30°, 60° H sin θ =

$$\frac{R}{2} \sin \omega/R \text{ or } \frac{\sqrt{3}}{2} \frac{R \sin \omega}{R} \text{ ie. } \frac{\sin \omega}{2} \text{ or } \sqrt{3}/2 \sin \omega.$$

Evidently the sum of the two Valanas Āyana and Ākṣa cannot be 90° as is also vouchasafed from geometry. So, here also, the flaw is evident.

Note 1. Srīpatyāchārya also followed Lallāchārya vide verses 18, 19, 20 Chandragrahaṇādhyāya, Siddhānta Sēkhara. It will be noted that the commentator of Siddhānta Sēkhara, while reiterating Bhāskara's stand as the correct one, himself commits a mistake in saying

In fact

$$\sin \xi = \frac{\sin \phi \sin h}{\cos \mu} = \frac{\sin \phi \sin z}{\cos \delta}$$ as proved by us before.

The commentator cited above overlooked that sin ξ could be also equal to $\frac{\sin \phi \sin h}{\cos \mu}$, wherein natakāla also is implied.

Note 2. It will be noted that even Pṛthūdakāchārya, while commenting on Brahmasphuta Siddhānta, ignored Brahmagupta and followed Lallācharya blindly.

Note 3. The formula given by Lallāchārya and followed by Pṛthūdaka as well as by Srīpati is very rough besides containing the flaw cited, in as much as both μ and δ are taken to be zero, which are not so.

SURYAGRAHANĀDHIKĀRA

Verse 1. In as much as the observer situated on the surface of the Earth and as such elevated by the radius of the Earth from the centre there of, perceives not the Sun and the Moon having the same longitude at the moment of conjunction, to be in the same line of sight, heyt being depressed unequally having different orbits, so I proceed to elucidate what are called Lambana and Nati ie. parallax in longitude and latitude, on which account they are not in the same line of sight.

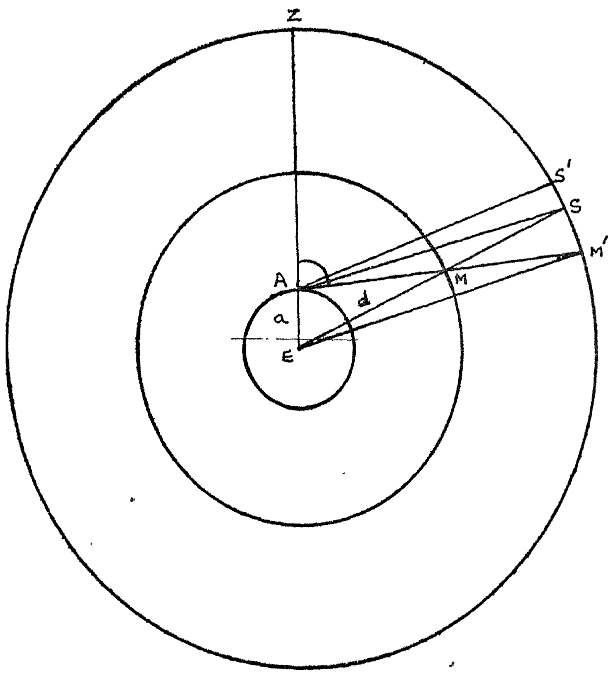


Fig. 91

Comm. (Refer fig. 91) Let E be the centre of the Earth, M and S the centres of the discs of the Moon and the Sun. Let A be the position of an observer on the surface of the Earth, elevated by the radius EA from E. Let M and S be in the same line of sight as seen from E. But as seen from A, AS and AM are respectively the lines of sight to the Sun and the Moon. Evidently these lines of sight differ the Moon being depressed more than the Sun. If a line AS′ be drawn which is parallel to the central line of sight namely EMS, we find that the Sun is depressed by the angle S′AS whereas the Moon is depressed by the angle S′AM′. These angles differ because the orbits of the Sun and Moon differ.

Here the angle S′AS will be very very small, its magnitude being in truth just about 8″ only. But the angle S′AM′ will be sufficiently large since the Moon is very near the Earth compared with the Sun. Taking ES and AS to be almost parallel due to the largeness of the Sun's distance, the angle $\widehat{SAM}$ will be almost equal to $\widehat{AMS}$ so that we could consider that the Moon is depressed from AS the line of sight to the Sun by the angle SAM′ = $\widehat{AME}$. This angle AME is called the geocentric parallax of the Moon and the angle $\widehat{ASE}$ that of the Sun $\widehat{M'AS}$ = angle of depression of the Moon over and above that of the Sun = M′AS′ − M′AS = $\widehat{EMA} - \widehat{ESA}$ = geocentric parallax of the Moon minus geocentric parallax of the Sun.

Verse 2. The presence or absence as well as the positiveness and negativeness of the parallax in longitude.

Compute the Lagna at the moment of conjunction of the Sun and the Moon. There will be no parallax in longitude when the Sun is situated at the point called Vitribha or the point whose longitude is $\equiv L-90°$, L being the longitude of the Lagna ie. the ascendant which is the point of intersection of the Ecliptic with the horizon. If the Sun's longitude falls short of the longitude of the Vitribha or exceeds it, there will be parallax in longitude which will be positive in the former case and negative in the latter.

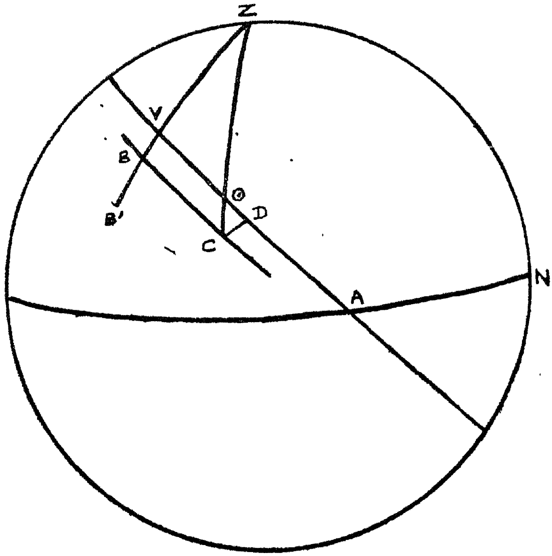


Fig. 92

Comm. (Ref. fig. 92) Let SN be the horizon, Z the zenith and VA the Ecliptic. A is the ascendant or Lagna. Let V be the point called Vitribha which is 90° behind A. Strictly speaking V is called Vitribhalagna or lagna from which three Rās'is or 90° are subtracted (Bha=Rās'i. त्रिभिर्विरहितम् वित्रिभम्; वित्रिभम् च तत् लग्नम् च वित्रिभलग्नम् ie. a point whose longitude is got by subtracting three Rās'is from that of the Lagna). Let ZV be the vertical of V so that $\widehat{ZVA} = 90°$. It will be seen that AV = 90° as follows. Let A′ be the point where the Ecliptic intersects the horizon on the west. One will construe that the Ecliptic is bisected by the meridian; but it is not so. Spherical triangles AVZ and A′VZ being right-angled at

V are congruent because AZ = A′Z and ZV is common ∴ AV = VA′. But AV + VA′ = 180° because the Ecliptic and the horizon being two great circles, they bisect each other. Hence AV = 90°. Then a celestial body situated at V will be depressed along ZV the vertical, say, to a point B. Let ⊙ be any arbitrary position of the Sun; then ⊙ will be depressed along the vertical Z⊙, say, to a point C. Draw CD perpendicular on the Ecliptic. Then ⊙D is the component of the parallax ⊙C along the Ecliptic where as DC is its component perpendicular to the Ecliptic. Thus ⊙D is the parallax in longitude and DC is the parallax in latitude. The word 'Lambana' means etymologically लम्बते अनेनेति लम्बनम् ie. that amount by which the celestial body is depressed (along the Ecliptic). In Hindu Astronomy the word Lambana is applied to parallax in longitude alone whereas the word Nati is applied to parallax in latitude. Hence to translate Lambana as parallax alone is not correct. The word Drik-lambana is applied to mean parallax along the vertical, and the word Sphutalambana is occasionally used to connote parallax in longitude.

As Bhāskara rapidly comments on the verses in this Gaṇitādhyāya, he having dealt with the subject of parallax elaborately under the caption, Grahaṇa Vāsanā, in the Golādhyāya, to catch up his thought, we have to treat the subject first from the modern view point and then elucidate what he has said in the Golādhyāya, much matter of which is reiterated by him under the commentary here in the Gaṇitādhyāya.

(Ref. Fig. 91) From the △EAM,

$$\frac{\quad}{a} = \quad d \qquad d$$

the Earth and d the distance of the celestial body (here the Moon)

$\therefore$ $\sin \widehat{EMA} = \frac{a}{d} \sin \widehat{ZAM} = \widehat{EMA}$ expressed in radian measure since $\widehat{EMA}$ is very small

$\therefore$ $\widehat{EMA}$ (expressed in radian measure) $= \frac{a}{d} \sin z$ I where z is the apparent zenith-distance of the Moon ie. zenith-distance as seen by the observer (in contradistinction to the geocentric zenith-distance of the Moon namely $\widehat{ZEM}$). In particular, when $z = 90°$, $\widehat{EMA} = \frac{a}{d}$ which is the maximum parallax known as the horizontal parallax ie. the parallax when the Moon is situated on the horizon of the observer. Also the parallax is zero when M is situated at Z as is seen from formula I and as is rightly remarked by Bhāskara in the words '

In fig. 91, $\widehat{EMA}$ is the angle by which the line of sight of the observer namely AM is depressed from the geocentric line of sight EM. Since the plane of the paper represents a vertical through the Sun and the Moon, the depression of either the Sun or the Moon or the excess of the depression of the Moon over the Sun are all in the vertical plane. This depression is called Drik-lambana because it is a lambana or depression in the Drik-maṇḍala or vertical.

This Drik-lambana varies as sin z as is seen from formula I where *a*, and *d* may be taken to be constants. (Both *a* and *d* vary slightly *a* varying slightly from place to place on the Earth, the Earth being an oblate spheroid, and *d* varying from position to position of the Moon).

The maximum horizontal parallax is given by $\frac{a}{d}$ in radian measure which is equal to, according to Bhāskara's estimate $\frac{1581}{2} \times \frac{1}{51566} \times 3438 = 53'$ approximately. Its modern value is about $57'$ so that the Hindu estimate is not far from truth. Here 1581 and 51566 are the values of a and d in Yojanas according to Bhāskara.

The Hindu astronomers do not, however, proceed exactly as we have done in the para above to obtain the maximum horizontal parallax. Their treatment is a little different and is as follows. Whereas according to Modern astronomy $\widehat{EM'A}$ (fig. 93) is viewed as the horizontal

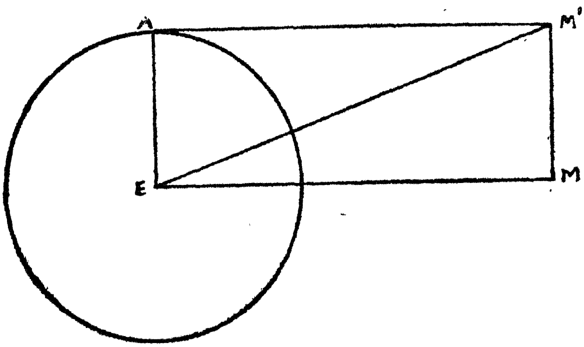


Fig. 93

parallax, in Hindu Astronomy $\widehat{MEM'}$ or the angular measure of the Moon's path equal to the radius of the Earth is taken to be the horizontal parallax. Both, of course, mean the same as is seen from the figure.

In as much as the Hindu astronomers knew very well what they term कलीकरण or converting linear distances into angular measure, converting a linear magnitude equal to the radius of the Earth namely MM′ at the lunar orbit into angular measure, they got

$\frac{MM'}{EM} \times 3438 \quad \frac{1581}{2} \times \frac{\;}{51566}$ radians $= 52'\text{-}42''$ as the maximum horizontal parallax.

Having got this estimate, they reckoned this angular measure in time as the time taken by the Moon to traverse the distance MM′ equal to the radius of the Earth as follows. The Moon traverses 790′-35″, which is exactly 15 times 52′-42″. Hence they said that the maximum horizontal parallax is $\frac{1}{15}$th of the Moon's daily motion in arc and expressing it in terms of time, that the maximum horizontal parallax is $\frac{1}{15}$ of a day or $\frac{1}{15}$th of 60 nādis or 4 nādis.

That this horizontal parallax is 4 nādis as a maximum, would have been also verified at the time of a solar eclipse when the Sun was situated on the horizon at the time of conjunction, by the fact that the eclipse occurred four nādis in advance of the moment of geocentric conjunction (which could be calculated very accurately by the Hindu astronomers, as could be seen by the very correct estimate of a lunation in Hindu Astronomy. In fact, the length of a lunation must have been estimated correctly by noting the time-interval between two solar eclipses or lunar and by dividing that time by the integral number of lunations elapsed in between the two eclipses).

The question then arises as to how the Hindu Astronomers could know the distance of the Moon. From the estimate of the horitontal parallax by actual observation, and from the geometry of fig. 93, a correct estimate of the distance of the Moon must have been arrived at.

Having thus known that the Moon traverses a distance equal to the radius of the Earth in 4 nādis, his daily linear motion was estimated to be 15 times the radius of the Earth ie. $\frac{15 \times 1581\frac{1}{3}}{2} = 11858\frac{3}{4}$ Yojanas.

The daily motion of the Moon having thus been estimated almost correctly, an act of inexpedience on the part of the Hindu astronomers was that they should have presumed that all the other planets including the Sun would be traversing the same linear distance during the course of a day. This led to a wrong estimate of the Sun's distance as well as his spherical radius. Also, they supposed wrongly that the parallax of the Sun also would be equal to $\frac{1}{15}$th of his daily arcual motion.

Their estimate of the spherical diameter of the Moon was, however, very near the truth, for, they argued, that if 790′-35″ angular motion per day corresponded to 11858$\frac{3}{4}$ Yojanas in linear measure, to what linear measure did the angular diameter of the Moon namely 32′-0″-9‴ correspond? The answer was

$$\frac{11858\frac{3}{4} \times}{790'\text{-}35''}$$

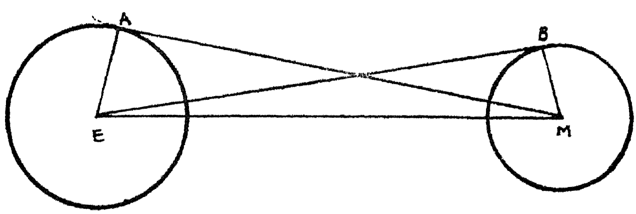


Fig. 94

It may be here pointed out that there is a relation between the angular radii of two celestial bodies as seen from each other and their mutual horizontal parallaxas (Fig. 94). Let E and M bet he centres of the Earth and the Moon respectively. $\widehat{EMA}$ = horizontal parallax of the Moon = angular radius of the Earth as seen from the Moon and $\widehat{BEM}$ = Angular radius of the Moon = Horizontal parallax of the Earth as seen from the Moon. Thus, we see that the Earth will be seen from the Moon, as a Moon with an angular radius equal to 57′. In other words our Earth will be a Moon to our Moon, having nearly 16 the area of our Moon's disc.

The periphery of the Moon's orbit was arrived at as follows. "If 790′-35″ of the Moon's angular motion corresponds to $11858\frac{3}{4}$ Yojanas to what periphery must $360 \times 60'$ correspond?" The answer is

$\frac{11858\frac{3}{4} \times 360 \times 60}{790'-35''} = 324000$ Yojanas. Reverting to the subject of parallax on hand, the Drik-lambana or the parallax along the vertical has the formula $\frac{4 \text{ H sin } z}{\text{R}}$ I

nādis in Hindu Astronomy where 4 nādis is the maximum parallax obtained when H sin z = R.

From fig. 92, $C\odot^2 = CD^2 + D\odot^2$ ie.

$$\text{Drik-lambana}^2 = \text{Nati}^2 + \text{Sphuṭalambana}^2 \qquad \text{II}$$

$$CD = \odot C \sin C\hat{\odot}D = \frac{4 \text{ H sin } z}{\text{R}}$$

$$\sin C\odot D = 4 \sin z \sin V\hat{\odot}Z$$

$$= 4 \sin \odot Z \sin V\hat{\odot}Z = 4 \sin ZV$$

Thus, the parallax in latitude at any point of the Ecliptic is that at the Vitribha which is conveyed by Bhāskara in the words "कक्षयोरन्तरं यत स्यात वित्रिभे सर्वं

$$\text{Also } \odot D = C\odot \cos C\hat{\odot}D = 4 \sin \odot Z \cos Z\odot V \frac{[\sin V\odot}{\text{R}^2} \qquad \text{IV}$$

= Maximum parallax × Vitribha-S'anku × H sine of the arc V⊙.

In the above working we proceeded in a modern way. It is worth-hearing Bhāskara as to how these results were arrived at elegantly and ingeneously from first principles.

In fig. 91, EMS is called Garbha-Sūtra whereas AS is called Dṛṣṭi-Sūtra.

ie. In as much as the Moon is depressed from the Dṛk-Sūtra, so this phenomenon goes by the name Lambana. It will be noted that in Hindu Astronomy geocentric parallax will not be treated separately for the Moon and the Sun but dealt with simultaneously as it is called for, in the context of a solar eclipse. They were interested in knowing the relative depression of the Moon with respect to the Sun rather than knowing the separate magnitudes with respect to the Moon and Sun, for which they had no application.

ie. In as much as the Garbha-Sūtra and Dṛk-Sūtra are identical in the direction EAZ (fig. 91) there is no parallax at the zenith.

Now consider the plane through ZV of fig. 92. Suppose the EMS of fig. 91 is in the direction EV. Then both the Sun and the Moon may be considered to have the same Vitribha at that moment of conjunction. Both the Sun and the Moon being then depressed along ZV, to V′ and V″ respectively the Ecliptic will then be a circle parallel to VA (fig. 92) through V′ and the orbit of the Moon will be another circle parallel to VA through V″, V″ being below V′. If we neglect, for a moment, the depression of the Sun, and consider VD to be the Ecliptic on which the Sun is situated undeflected, and BC to be the deflected orbit of the Moon relative to the Sun, then VB is the Nati of the Moon, which will be the same distance between VD and BC, ie. the orbits of the Sun and the Moon.

This fact was proved by us analytically in the modern way showing that CD = BV.

This Nati it is that influences the latitude of the Moon, which may cause apparent conjunction when there is no geocentric conjunction and which does not show an apparent conjunction when there is a geocentric conjunction. In other words parallax in latitude plays a very important part in solar eclipses. Also ⊙D being the parallax in longitude, the moment of apparent conjunction might be preceded or followed by a geocentric conjunction according as the Sun lies along VA or AV. Thus having determined the exact moment of apparent conjunction using tne magnitude of ⊙D, then we have to rectify the latitude using the magnitude of VB. If that rectified latitude of β falls short of $R+r$ where R is the angular radius of the Sun and r that of the Moon, then there will be a solar eclipse.

It will be noted that when the Sun coincides with V at the moment of conjunction, there is no parallax in longitude ⊙D being zero (Fig. 92) in that position. Also there will be no parallax in latitude when the Ecliptic assumes the position of a vertical circle passing through the zenith, the Drik-lambana then being entirely along the Ecliptic. In this case the Vitribhalagna V will coincide with Z and $\frac{4\ \text{H sin V} \odot}{\text{R}}$ which is termed the Madhyamalambana is now entirely along the Ecliptic and as such it is the Sphutalambana in this case. We have said above that when V coincides with Z, the Madhyamalambana is zero at Z, and that the maximum is equal to 4 nādis on the horizon. In between Z and the horizon it has the formula $\frac{4\ \text{H sin V} \odot}{\text{R}}$. Noting further that in this case when the H cosine of the zenith-distance of V ie. the S'anku of V is R, the entire lambana is along the Ecliptic, the nāti being zero, and the Madhyamalambana is itself the Sphuta-lambana, and again when V does not coincide with Z, H cos ZV is no longer R but has assumed Kōti-Rūpa ie. the form of a H cosine, as well as the Sphuta-lambana also, which assumes Kōti-Rūpa ie. of the form ⊙D of Fig. 92, where ⊙C is the Madhyamalambana, ⊙D is

Kōti = Sphuta-lambana, and DC = Bhuja = Nāti, it is argued that the Sphuta-lambana is proportional to H cos ZV, assuming a maximum value when V coincides with Z (⊙ not being at Z).

Verses 3 and 4. Parallax in longitude based on two proportions.

Compute the H cosine of ZV, by calculating the rising time of AV, the Kujyā, Dyujyā and Antyā pertaining to V, as was formulated in the Tripraśnādhikāra, then H sin V⊙, multiplied by 4 and divided by R, and again multiplied by H cos ZV and divided by R again gives the parallax in longitude.

Comm. As per the above formula, parallax in longitude equal to ⊙D of Fig. 92 is equal to

$$\frac{4 \text{ H sin V}\odot \times \text{H cos ZV}}{R^2}.$$

This is evidently derived out of two proportions that the parallax in longitude is proportional to H sin V⊙ as well as H cos ZV. This we have already derived through modern methods as formula IV.

Under verse 2. The two proportions are (1) V coinciding with Z, if by H sin V⊙ equal to R, we have 4 nādis as the maximum lambana on the horizon, what shall we have by an arbitrary H sin V⊙? The result is

$\frac{4 \text{ H sin V}\odot}{R}$ and (2) V not coinciding with Z, if by H cos ZV equal to R we have $\frac{4 \text{ H sin V}\odot}{R}$ as the Madhyamalambana, what shall we have for an arbitrary H cos ZV? The result is

$$\frac{4 \text{ H sin V}\odot}{R} \times \frac{\text{H cos ZV}}{R}$$ as formulated.

First half of verse 5. Alternate method of rectifying lambana. The Madhyamalambana multiplied by 12 and divided by the Chāyākarṇa of the Vitribha will also give the Sphuta-lambana.

Comm. From Tripras'nādhikāra, we have

$\frac{12}{K} = \frac{H \cos Z}{R}$ so that in the formula cited above instead of

$\frac{H \cos ZV}{R}$ we are asked to use 12/K.

Latter half of verse 5 and first half of verse 6.

$$\text{Dṛk-nati}^2 = H \cos^2 ZV - H \cos^2 Z\odot$$

$$= H \sin^2 Z\odot - H \sin^2 ZV \text{ (fig. 92)}$$

$$\frac{4 \text{ Dṛk-nati}}{R} = \text{Sphutalambana.}$$

Comm. We shall first prove this on modern lines.

$$\text{Cos } Z\odot = \cos ZV \cos Z\odot$$

$$\therefore \cos^2 ZV - \cos^2 Z\odot = \cos^2 ZV (1 - \cos^2 V\odot)$$

$$= \cos^2 ZV \sin^2 V\odot = \frac{H \cos^2 ZV\, H \sin^2 V\odot}{R^4}.$$

Also $\cos^2 ZV - \cos^2 Z\odot$

$$= \sin^2 Z\odot - \sin^2 ZV =$$

$$\frac{H \cos^2 ZV - H \cos^2 Z\odot}{R^2} = \frac{H \sin^2 Z\odot - H \sin^2 ZV}{R^2}$$

$$\therefore H \cos^2 ZV - H \cos^2 Z\odot = H \sin^2 Z\odot - H \sin^2 ZV$$

$$= \frac{H \cos^2 ZV\, H \sin^2 V\odot}{}$$

$$\therefore \text{ Dṛk-nati defined above} = \sqrt{H \cos^2 ZV - H \cos^2 Z\odot}$$

$$\overline{\sin^2 Z\odot - H \sin^2 ZV} = \frac{H \cos ZV\, H \sin V\odot}{R}$$

$$\therefore \frac{4 \text{ Dṛk-nati}}{R} = \frac{4 H \cos ZV\, H \sin V\odot}{R^2}$$

$= \text{Sphutalambana.}$

Bhāskara's proof proceeds in two stages from first principles. In the first place when V coincides with Z, Lambana is seen to be equal to 4 nādis on the horizon and zero at Z ie. it is zero when H sin Z⊙ = 0 and 4 nādis, a maximum when H sin Z⊙ = R. So, it is meet that Lambana should be taken to be proportional to H sin Z⊙ ie. proportional to natajyā. In this context the lambana termed as Madhyamalambana is entirely along the ecliptic. It is taken to be in the form of Karṇa, because in the position of ⊙C also it is in the form of a Karṇa.

Then let the Ecliptic be deflected from the zenith (deflected = क्षिप्त). Vitribhalagna then being deflected from the position of Z, occupies the position of V (fig. 92). So ZV is called Dṛk-kshepa since the Ecliptic which was in the form of a Dṛk-mandala is deflected from that position. Also the circle ZV is called Dṛk-kshepa-mandala because V is deflected along that circle. Now consider the △ whose sides are H sin ZV, H cos ZV and R. H cos ZV equal to R and as such in the form of a Karṇa corresponds to the Madhyamalambana which is also in the form of a Karṇa; when this Vitribha-S'anku assumed the form H cos ZV, ie. rendered a Kōti from its form of a Karṇa, R, the Sphutalambana is also rendered a Kōti in the form of ⊙D so that

$$\frac{\text{Madhyamalambana}}{R} = \frac{\text{Sphutalambana}}{H \cos ZV}$$

$$\text{Sphutalambana} = \frac{H \cos ZV}{R} \text{ Madhyamalambana}$$

$$ZV \times 4\ H \sin Z\odot$$

Then Bhāskara says that we could look at this, from another angle in the words "यदेव स्फुटलम्बनस्य कोटिरूप

In fig. 92, H sin Z⊙ is in the form of a Karṇa; H sin ZV is in the form of the corresponding Bhuja. This triangle formed by these two as sides may be taken to be similar to the triangle ⊙DC, both being called parallax △s. This plane triangle ⊙DC is like the plane triangle which has for its sides H sin λ, H sin δ, where λ and δ are the longitude and declination of a point of the Ecliptic.

In the Tripras'nādhikara, we had occasion to deal with this triangle and there we had R

$$\frac{\sin^2 \lambda - H \sin^2}{H \cos \delta} \quad - \qquad \text{right}$$

sion of the point. Similarly R $\dfrac{\sqrt{H \sin^2 Z\odot - H \sin^2 ZV}}{H \cos ZV}$

= H sin V⊙. In other words Dṛk-nati is H sin V⊙ projected into a circle of radius H cos ZV from a circle of radius R. We have the proportion

$$\therefore \frac{C\odot}{H \sin Z\odot} = \frac{CD}{\odot \sin ZV} = \frac{D\odot}{\sqrt{H \sin^2 Z\odot - H \sin^2 ZV}}$$

$$= \frac{D\odot = \text{Sphutalambana}}{\sqrt{H \sin^2 Z\odot - H \sin^2 ZV}}$$

The quantity under the radical in the denominator is called Dṛk-nati for the following reasons.

When V coincides with Z, V⊙ is the Dṛk-mandala-nata, so that when V is deflected also, we continue to view the Dṛk-nati placed along V⊙. Since Madhyama-lambana ⊙C is in the form of a Karṇa in the △⊙DC, we perceive it to be in the form of a Karṇa even when V coincides with Z. This Madhyamalambana being equal to Sphutalambana when V coincides with Z, Sphuta-lambana is also in the form of a Karṇa then. Now in the position ⊙DC, Sphutalambana has assumed the position of a Kōti ie. the Sphutalambana which, in the form of a Karṇa, being placed along Dṛk-mandala natāmsa, is now rendered a Kōti and is placed along V ⊙, So the quantity $\sqrt{H \sin^2 Z\odot - H \sin^2 ZV}$ which is the Kōti of the △ formed by H sin Z⊙ and H sin ZV, corresponds to the Kōti of Sphuta-lambana ⊙D. So we call

$\overline{\sin^2 Z\odot - H \sin^2 ZV}$ as Dṛk-nati, in as much as the Sphutalambana being placed along V ⊙ in the form of a Karṇa when V ⊙ is Dṛk-mandala-nata, continues to be placed along V ⊙ in the deflected position also and becomes a Kōti corresponding to the Kōti of the triangle formed by H sin Z⊙ and H sin ZV ie. corresponding to the quantity $\sqrt{H \sin^2 Z\odot - H \sin^2 ZV}$. The Sphuta-lambana should be construed as being associated with Dṛk-mandala-nata which term is now abbreviated to the term Dṛk-nati.

At A of fig. 92, the Dṛk-nati $= \sqrt{R^2 - H \sin^2 ZV}$ $= H \cos ZV = H \cos ZV =$ Vitribha-lagna-S'anku. Hence the proportion proceeds in accordance with this Dṛk-nati.

Verse 6 (*latter half*) *and first half of verse* 7. Alternative method of obtaining parallax in longitude.

$$\left(\frac{H \cos ZV}{/4}\right)^2 - \left(\frac{H \cos Z\odot}{R/4}\right)^2$$

or

$$\left(\frac{\quad}{R/4}\right) - \left(\frac{\overline{V}}{R/4}\right)^2$$

gives the parallax in longitude expressed in nādis.

Comm. These formulae just constitude another mode of expressing the parallax in longitude and the equivalence of the formulae with the formula $\frac{4}{R}$ Dṛk-nati is evident.

Latter half of verse 7. Use of the parallax in longitude.

The time of the ending moment of New Moon ie. the of geocentric conjunction is to be rectified by this parallax in longitude to get the moment of apparent conjunction by the method of successive approximation.

Comm. In as much as the moment of apparent conjunction for an observer situated somewhere on the surface of the Earth precedes or follows the moment of geocentric conjunction being preponed or belated by the parallax in longitude, we have got to take this parallax in longitude into account and compute the moment of apparent conjunction. This computation has to proceed according to the method of successive approximation since the hourly motions of the Sun and the Moon vary as well as the parallax in longitude. When the Sun is in advance of V, the Sphutalambana advances the Moon more than the Sun so that the moment of apparent conjunction is past. Hence the correction is negative and vice versa.

Verses 8 and 9. Computation of the parallax in longitude without an appeal to the method of successive approximations.

Let Para $= \frac{13}{32}$ H cos ZV ; $\{\text{Para} \sim \text{H sin} \odot \text{L}\}^2 +$

$\text{H cos}^2 \odot \text{L} = \text{K}^2 \;\; \text{H sin}^{-1} \left\{ \frac{\text{H cos} \odot \text{L} \times \text{Para}}{\text{K}} \right\} =$

parallax in longitude.

Ref. fig. 95, $E_1 E_2$ is taken to be what is termed Para equal to $\frac{4}{R}$ H cos ZV, H cos ZV being the Vitribha-S'anku.

Since $\frac{4}{R}$ could be written as $\frac{\text{H sin 24}}{\text{R}}$, since 4 ghatis $=$

$\frac{4 \times 360}{60} = 24°$, 60 ghatis being equivalent to 360°,

$\frac{4}{R}$ H cos ZV $= \frac{\text{H sin 24}}{\text{R}} \times$ H cos ZV. Imagining for a moment H cos ZV has come in the place of H sin λ

pertaining to the formula $H \sin \delta = \frac{H \sin \lambda \times H \sin 24}{R}$

$\frac{4}{R}$ H cos ZV = Para = the H sine of the declination of that point whose longitude is equal to Vitribha-S'anku. In other words Para is termed as the Vitribha-S'anku-Rūpa-Krānti-Vṛttiya-Bhujajyājanita-Krāntijyā.

Now take $E_1 E_2$ = Para defined above. Draw circles of equal radii with E_1 and E_2 as centres. Call (E_1) and (E_2) as the Chandra-Kakshāmandala and Ravi-Kaksha Mandala. Para by its formulation as $\frac{4}{R}$ H cos ZV, is equal to the maximum parallax in longitude for a given H cos ZV ie. for a given position of V with respect to Z. This being so, the parallax in longitude for an arbitrary position of ⊙ with respect to V will be

$\frac{\text{Para} \times H \sin (\odot - v)}{R}$ according to the previous lation thereof. This form of the formula by its similarity with the formula $\frac{a}{R}$ H sin m, pertaining to the eccentric-circle-theory, suggested to Bhāskara that the parallax in longitude could be derived from the theory of the eccentrics or Prati-Vṛtta-Bhangī. In fig. 95, it will be noted that E_1, E_2 are not the centre of the Earth and the position of the observer on the surface of the Earth but such points as $E_1 E_2$ is made equal to $\frac{4}{R}$ H cos ZV or $\frac{a}{d}$ H cos ZV of the modern figure $\frac{4}{R}$ being equal to $\frac{a}{d}$, so that $E_1 E_2$ is of a variable magnitude varying with H cos ZV.

Comm. When H cos ZV = R ie. when V coincides with Z, we have the maximum parallax. What then will

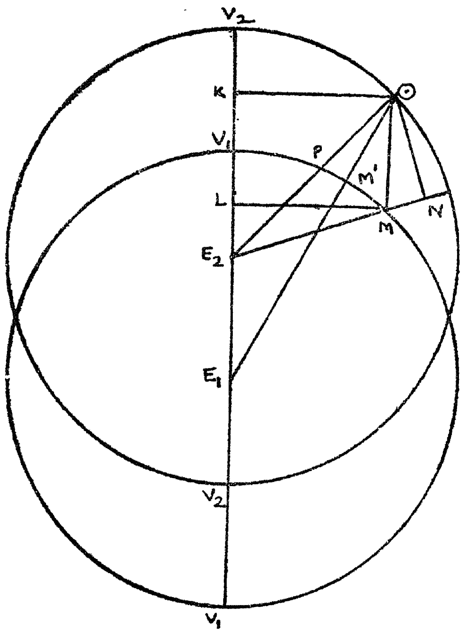


Fig. 95

be had for an arbitrary H cos ZV? The result is

$\frac{\text{H cos ZV}}{3438} \times$ H sin 24 since 4 nādis correspond to 24°, 60 nādis corresponding to 360°. Hence the result is

$\frac{\text{H cos ZV} \times 1397}{3438}$. Converting $\frac{3438}{1397}$ into a continued fraction we have $2 + \frac{1}{2+}\frac{1}{5+}\frac{1}{1+}\frac{1}{9}$ ......of which the convergents are $\frac{2}{1}$, $\frac{5}{2}$, $\frac{27}{11}$, $\frac{32}{13}$ and $\frac{32}{13}$ is a very good convergent preceding a large quotient namely 9. So the result may be written as $\frac{13 \text{ H cos ZV}}{32}$ which is symbolized as

Now parallax in longitude $= \frac{\text{Para} \times \text{H sin} (\odot - v)}{\text{R}}$.

When H sin $(\odot - v) =$ R, the parallax will be equal to

Para. This formula by its similarity with the formula pertaining to the eccentric theory led Bhāskara to use the method of eccentric circles to obtain the parallax. It is indeed ingenious on his part to have conceived the applicability of that method.

Further it is rather curious that 4 nādis of the maximum parallax should correspond to 24°. This also led Bhāskara to conceive similarity between the formulae $H \sin \delta = \frac{H \sin \lambda \times H \sin 24°}{R}$ (the formula used to obtain the declination δ given the longitude λ of a point of the Ecliptic) and the formula $\frac{H \cos ZV \times H \sin 24}{R}$ $=$ Para. So from an arbitrary $H \sin \lambda$ equal to Vitribha-S'anku, Para is derivable as $H \sin \delta$. In other words Para is called Vitribha-S'anku·Rūpa-Krānti·Vṛttiya Bhujajyā-Janita-Krāntijyā.

Now the doubt arises, namely that when the formula longitudinal parallax $= \frac{\text{Para} \times H \sin(\odot - v)}{R}$ resembles the formula $\frac{a}{R}$ $H \sin m$ which pertains to the Equation of centre, why does Bhāskara suggest that the parallax is derivable without the application of the method of successive approximations, by appealing to the method S'īghraphala. The doubt is here two fold (1) where is the necessity for the method of successive approximation to obtain the parallax, though it be called for, to obtain the moment of conjunction? (2) why does Bhāskara appeal to S'īghrakarma and not Mandaphala, when the formula suggests the latter, by the presence of R and there is no K at all?

The answer is as follows. In the first place, even in the modern formula for parallax namely $a/d \sin z$, Z is the zenith-distance pertaining to the observer and not the geocentric zenith-distance, which are respectively called pṛṣṭhīya and garbhīya natāṃsas. Also the parallax is the angle between the geocentric direction of the Moon and that of the observer. (Vide fig. 91 where parallax = $\widehat{EMA}$).

In deriving this parallax, we are using the apparent zenith-distance of the Moon and not the geocentric zenith-distance of the Moon. In fig. 92 the position of $\odot$ corresponds to the geocentric position, whereas D corresponds to the position of the observer on the surface of the Earth. So, as we use the apparent zenith-distance as argument to obtain parallax along the vertical, so we have to use, VD as the argument to derive the parallax in longitude and not $V\odot$. So, the method of successive approximations is called for as $\odot D$ is first computed from the argument $V\odot$ and VD is to be made the argument thereafter. This means that $V\odot$ may be construed as Madhyakēndra and VD as Sphutakēndra. Now applying this idea to fig. 95, $V_2 E_2 \odot$ may be construed as Sphutakēndra whereas $V_2 E_1 \odot$ may be construed as Madhyakēndra.

From the similarity of the triangles $\odot NM$, and

$$E_2 LM, \frac{\odot N}{LM} = \frac{\odot M}{E_2 M} \quad \therefore \quad \odot N = \frac{\odot M}{E_2 M} \times LM =$$

$$\frac{\text{Para}}{K} \times \odot K = \frac{\text{Para}}{K} \times H \sin \widehat{KE_2 \odot}$$

where E_2M is termed the Karṇa and $\widehat{KE_2\odot}$, the Sphutakēndra is made the argument. Thus parallax in longitude which was originally formulated as $\frac{\text{Para} \times H \sin \overline{\odot - v}}{R}$ (in which case, the method of successive approximation was called for), is now formulated as $\frac{\text{Para}}{K} \times H \sin (KE_2 \odot)$

where that method of successive approximation is circumvented and where by the presence of K in the place of R, analogy is with the eccentric method of formulation of S'ighraphala and not that of Mandaphala. Also

$$K^2 = E_2M^2 = E_2L^2 + ML^2 = (E_2K - LK)^2 + \odot K^2 = (E_2K - M\odot)^2 + \odot K^2 = (H \cos \widehat{KE_2\odot} - \text{Para})^2 + H \sin^2 \widehat{KE_2\odot}.$$

But if L be the lagna of the moment $L\odot = 90 - V\odot$ so that $H \cos \widehat{KE_2\odot} = H \sin \odot L$ and $H \sin \widehat{KE_2\odot} = H \cos \odot L$ $\therefore$ $K^2 = (H \sin \odot L - \text{Para})^2 + H \cos^2 \odot L$ as formulated in the verse.

Fig. 95 is in the plane of the Ecliptic. The parallax in the vertical circle is projected on to the plane of the Ecliptic by taking $\frac{4}{R}$ H cos ZV as the Para, and deriving the parallax in longitude from this Para.

Now, the doubt arises as to why the S'ighroccha is not taken to coincide with the Vitribha but is taken as removed 180° therefrom.

Verse 10. H sin ZV (of fig. 92) is called the Dṛk-kshepa of the Sun, which is considered to be north in case the northern declination of the Vitribha is greater than ϕ the latitude, otherwise south.

Comm. Let in fig. 96, AV be the Ecliptic whereof A is the ascendant or Lagna and V the Vitribhalagna. Let EQR be the celestial Equator. Let δ be the declination of the Vitribhalagna. Then if $\delta > \phi$. then ZV, the arc of the the Dṛk-kṣepa, (H sin ZV being defined as the Dṛk-kṣepa) as well as H sin ZV are considered to be north. Thus in fig. 96, it is north whereas in fig. 97 it is south. (In fig. 97, *r* is shown outside the celestial sphere, signifying that r is in the western hemisphere and is brought into view for clarity).

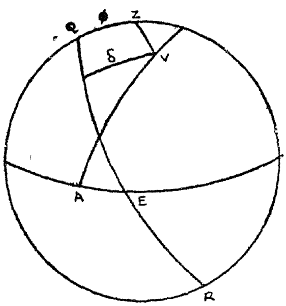


Fig. 96

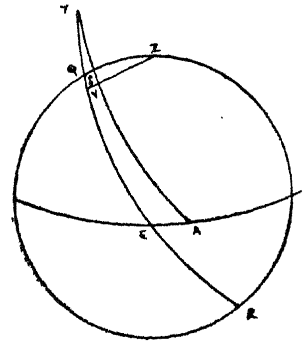
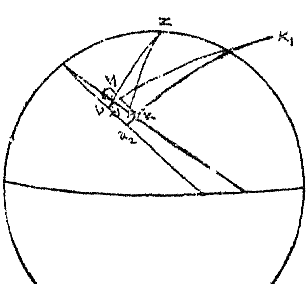


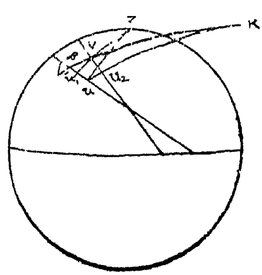
Fig. 97

Verse 11 *and first half of verse* 12. Then the sum of ZV and the latitude of V assuming V to be the Moon, or the difference of the above two, as the case may be, according as both of them are north or of opposite directions, gives the arc whose Hsine is the Dṛk-kṣepa of the Moon, The Dṛk-kṣepas of the Sun and the Moon multiplied respectively by $\frac{1}{15}$th of their daily motions and divided by the radius R (equal to 3438′) are the parallaxes of the Sun and the Moon in latitude. The sum or difference of these parallaxes according as they are of opposite or the same direction, is the true parallax in latitude in the context of a solar eclipse.

Comm. The true parallax in latitude sought above is the relative parallax of the Sun and the Moon in latitude. Suppose in fig. 92, VB is the parallax in latitude pertaining to the Sun and VB′ that pertaining to the Moon; then BB′ is the relative parallax, the difference being taken in this case because both are of the same direction.

Parallax in latitude namely CD in fig. 92, we saw equal to VB which is equal to $\frac{4H}{R}$ sin ZV, In other words the parallax in latitude either of the Sun or the Moon is equal to $\frac{4}{R}$ H sin of the zenith-distance of the respective Vitribhalagna wherever the Sun or the Moon be situated in their orbits namely the Ecliptic or the Vimandala. Let H sin ZV be the Dṛk-kṣepa of the Sun, V being the Vitribhalagna pertaining to the Sun and let H sin Zv be the Dṛk-kṣepa of the Moon where v is the Vitribhalagna of the Moon. (Ref. figures 98 and 99) Let K′ be the pole of the Vimandala and vv_2 the latitude of v. Since v and V are in the proximo, the latitude of v may be taken to be very nearly equal to the latitude of V so that ZV $\pm$ latitude of V is very nearly equal to Zv. In fig. 98, ZV—latitude of V is very nearly equal to Zv because both ZV and latitude of V are of the same direction. In fig. 99 ZV+latitude of V is very namely equal to Zv because both arc of opposite direction. Thus Zv = ZV $\pm$ latitude of V approximately and H sin ZV and H sin Zv are the Dṛk-kṣepas of the Sun and the Moon respectively. Having got these Dṛk-kṣepas $\frac{4}{R}$ $\times$ Dṛk-kṣepa gives the nati in each case ie. the parallax in latitude and the sum or difference of these natis as mentioned in the beginning of the commentary of this verse gives the relative parallax of the Moon with respect to the Sun which is called the





true parallax in latitude. This true parallax in latitude increases or decreases the latitude of the Moon at the moment of conjunction as is going to be mentioned in the latter half of verse 14.

In deriving the parallax in latitude from the respective Dṛk-kṣepas, instead of using the formula $\frac{4}{R}$ Hsine (Dṛk-kṣepa) which is an expression in time, it is sought to express the same in arc because the latitude of the Moon is expressed in arc and we have to take the sum or difference of the latitude and the parallax in latitude to obtain the apparent latitude of the Moon at the moment of conjunction. In the case of the parallax in longitude we sought to express the same in time because the moment of apparent conjunction was sought therefrom.

Latter half of verse 12 *and first half of verse* 13. An approximate method of obtaining the relative parallax in latitude of the Moon with respect to the Sun.

The Hsine of the zenith-distance of the nonagesimal pertaining to the Moon or what is called the Moon's Dṛk-kṣepa multiplied by 2 and divided by 141, gives the relative parallax in latitude of the Moon with respect to the Sun; or working with the smaller table of Hsines (where R is taken to be 120) the Moon's Dṛk-kṣepa being multiplied by 2 and divided by 5 and the result being increased by $\frac{1}{60}$th of itself gives approximately the relative parallax in latitude.

Comm. Herein, the Vitribha or the nonagesimal of the Sun is taken to concide with that of the Moon. In other words the Dṛk-kṣepas (the Hsines of the zenith-distances of the nonagesimals, of both the Sun and the Moon are taken to be identical. Then using the following proportion "If by a Dṛk-kṣepa equal to R, the relative parallax in latitude is equal to $\frac{1}{15}$th of the difference of the daily motions namely 48′–46″, what will it be for an arbitrary Dṛk-kṣepa ?" We have

continued fraction, this will be equal to

$\frac{1}{70+}\frac{1}{1+}\frac{1}{1+}\frac{1}{10+}\frac{1}{3}$ of which a very approximate convergent is $\frac{2}{141}$ as taken by Bhāskara.

If the radius be taken to be 120, the coefficient of D will be $\frac{48\frac{3}{4}}{120}=\frac{195}{480}=\frac{13}{32}=\frac{1}{2+}\frac{1}{2+}\frac{1}{6}=\frac{2}{5}$ very approximately.

Latter half of verse 13 *and first half of verse* 14. An easy method to compute the parallax in longitude and latitude.

Taking the Dṛk-ṣepa of the Moon as well as the Sun to be the Hsine of the meridian zenith-distance of the Vitribha and the H cosine of its meridian zenith-distance as the Vitribha-S'anku, the parallaxes in latitude and longitude could be got from them respectively.

Comm. Parallax in longitude is computed from the Vitribha-S'anku, whereas parallax in latitude is computed from the Dṛk-kṣepa or the Hsine of the zenith-distance of the Vitribha. Thus for both the purpose the Vitribha's position is important, whose zenith-distance and altitude give respectively the parallax in latitude and longitude. Since in practice it is a little cumbrous to obtain the Vitribha's altitude and zenith-distance, an approximate procedure is suggested. Obtaining the declination or the Sphuta-krānti of the Moon taking him to coincide with the Vitribha by the method described in verse 3 of the Graha-cchāyādhikāra, and using the formula $z+\delta=\phi$, the meridian zenith-distance of the Vitribha can be got. This may be assumed to be the Dṛk-ṣepa approximately. The complement of the meridian-zenith-distance may be assumed to be the Vitribha-S'anku approximately. Then the parallaxes in latitude and longitude could be computed respectively from the two as described before.

The following figure gives a particular nomenclature that was in the mind of Kamalākara, the author of Siddhāntatattvavivēka.

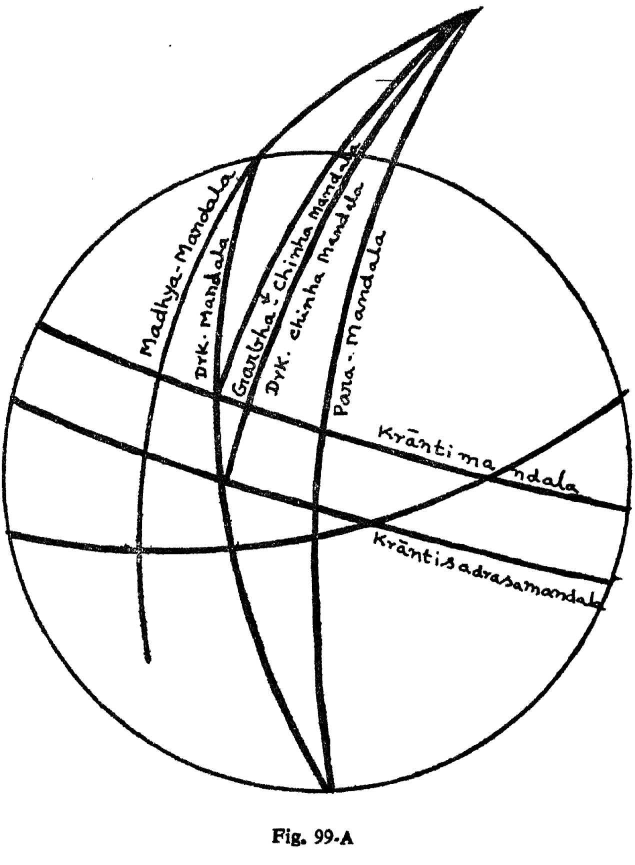


Fig. 99-A

Latter half of verse 14. The purpose of obtaining the parallax in latitude.

The apparent latitude of the Moon is equal to the algebraic sum of his geocentric latitude and the parallax in latitude. From this apparent latitude are to be calculated the Sthiti-khanda and Marda-khanda of the solar eclipse (by the method described in the chapter on lunar eclipse, taking the eclipsing body or grāhaka to be the Moon and the eclipsed or Grāhya to be the Sun).

Comm. The geocentric parallax of the Moon has a double effect on the occurence of a solar eclipse as mentioned before. If the parallax be resolved along the Ecliptic and along a secondary to the Ecliptic, we have respectively the parallax in longitude and that in latitude. The parallax in longitude makes the apparent moment of conjunction at a given place, differ from the moment of the geocentric conjunction, whereas the parallax in latitude makes the magnitude of apparent latitude of the Moon at the place of observation differ from that of the geocentric. The apparent latitude is equal to the sum or difference of the geocentric latitude and the parallax in latitude. Having got the apparent latitude, the computation of the Sthiti and Marda-khandas could be done according to the method described in the chapter on lunar eclipse.

For a point C on the surface of the Earth, a solar eclipse occurs if the latitude of the Moon be less than MD

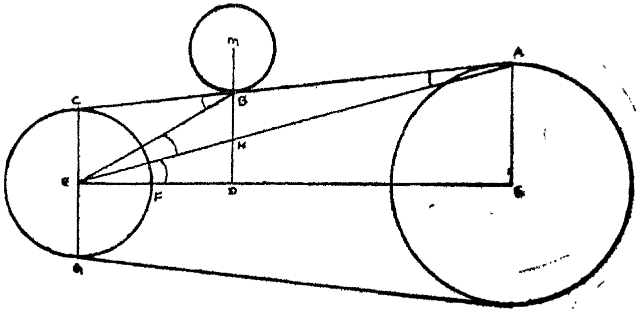


Fig. 100

(vide fig. 100) where M is the centre of the Moon and MD the latitude of the Moon at the point of first contact

$MD = MB + BD = m + \widehat{BED}$ where m is the semi-diameter of the Moon's disc. But

$\widehat{BED} = \widehat{BEA} + \widehat{AES} = \widehat{CBE} - \widehat{CAE} + \widehat{AES} = P - p + s$ where P and p are the parallaxes of the Moon and the Sun and s the angular semi-diameter of the Sun. Thus in order that a solar eclipse may be possible for some point of the Earth, the latitude of the Moon at the moment of conjunction must be less than $P + s + m - p$ $= 57' + 16' + 15' = 89'$ approximately. The lesser the northern latitude of the Moon at the moment of conjunction, more places situated on the surface of the Earth between C and F will have solar eclipse where F is the sub-solar point ie. the point of the Earth which has the Sun in the zenith at the time of conjunction. Similarly, if the southern latitude of the Moon is less than 89′ at the moment of conjunction the places situated on the Earth between G and F will have solar eclipse. In particular the sub-solar point F will have solar eclipse if the latitude of the Moon at the moment of conjunction is less than HD ie. less than $s+m$ ie. 33′ approximately. The sub-solar point will have no parallax, so that the terms P and p in $P+s+m-p$ vanish. For the other points ie. points between F and C or G parallax will be there and the latitude may be greater than $s + m$ but less than ’$-p$ to have an eclipse. A latitude of 33′ corresponds to a distance of $\frac{33 \times 15}{70} = \frac{99}{14} = 7\frac{1}{14}^{\circ}$ of the Sun with respect to a node. Thus if at the moment of conjunction, the latitude of the Moon be less than 7°, even the sub-solar point must have an eclipse.

Verses 15, 16, 17. To find Sparśakāla, Mokṣakāla, Sammīlanakāla and Unmīlanakāla.

First compute the time called Sthiti-khanda (as mentioned in the chapter on lunar eclipses). The ending moment of local Amāvāsyā or what is called the moment of local conjunction is known as the Madhya-Grahakāla or the moment of the middle of the eclipse. Subtract the Sthiti-khanda from the computed time of Geocentric conjunction; the result will be the approximate Spars'a-kāla. This has to be rectified for parallax in longitude as well as the approximate Madhyagrahakāla of geocentric conjunction to obtain the local Spars'akāla and the local Madhyagrahakāla; Similarly the Mokṣakāla, the Sammīlana and the Un-mīlanakālas are to be rectified for parallax in longitude. But while effecting this correction for the parallax in longitude, the Moon's latitude also differs for the corrected time which in turn effects the durations of Sthiti-khanda, Mokṣa-khanda etc. Correcting the first computed Sthiti-khanda. Mokṣa-khanda etc. for this variation in the latitude, and subtracting the Sthiti-khanda from the time of Madhya-graha, we have a better approximation for the Spars'akāla. In as much as parallax in longitude, that in latitude, and the Moon's latitude vary from time to time, and the times of Spars'a, Madhyagraha etc. are effected by them, the process of computation proceeds by the method of successive approximation. Subtracting the rectified Marda-khanda from the rectified Madhyagrahakāla, we have the true Sammīlanakāla; similarly adding the former to the latter we have the true Un-mīlanakāla.

If, as mentioned before in verse 9, the parallax in longitude is found without using the method of successive approximation, the Spars'a-kāla and the Mokṣa-kāla are had at once. But the latitude of the Moon and the parallax in latitude are to be computed using the then longitudes of the Moon and the non-agesimal.

Comm. Clear.

Verses 18, 19. To obtain the true values of the Bhuja and the Iṣṭakāla.

The remaining work proceeds on the lines indicated in the chapter on 'lunar eclipses' (ie. the computation of the Bimbavalana, Bhuja, Koti and the like is to be done as indicated there). The Bhuja will be rectified by multiplying it by the Sthiti-khanda obtained by adopting the latitude of the Moon effected by parallax in latitude and divided by the Sthiti-khanda rectified for parallax in longitude. Similarly given the grāsa ie. the magnitude of the eclipse, the result found before by verse 15 in the chapter of lunar eclipses, is to be multiplied by the Sthiti-khanda rectified for parallax in longitude and divided by that obtained adopting the latitude of the Moon effected by parallax in latitude, and the result so obtained being subtracted from the Sthiti-khanda, we get the Iṣṭa-kāla.

Comm. Refer fig. 73. The Sthitikhanda is the time taken by the centre of the eclipsing body to go from C_1 to N relative to the eclipsed body, ie. keeping the eclipsed body fixed. The Bhuja at any intermediate point of time between the moment of first contact and the middle of the eclipse is NC of fig. 73; and the Iṣṭakāla is the time elapsed between the moment of first contact to the moment when the centre of the eclipsing body occupies any arbitrary position C. In the context of the lunar eclipse, the Bhuja was calculated by the formula (T—I) (m_1-s_1) where T is the Sthiti-khanda, I = Iṣṭakāla, m_1 and s_1 the motions of the Moon and the Sun on the day concerned. This Bhuja may be also expressed in the form $\sqrt{(R+r-g)^2-\beta^2}$ where R, r are the radii of the eclipsing and eclipsed bodies, of the grāsa and β the latitude of the Moon at the middle of the eclipse. Given the grāsa G, the formula to find I the Iṣṭakāla, is

$$\frac{\quad g)^2-\beta}{m_1-s_1}.$$

In the present context of the solar eclipse, β, the latitude of the Moon is effected by the parallax in latitude, so that it is a variable. We are to make a correction for this variability, both in the computation of Sthiti-khanda, Bhuja and Iṣta-kāla. The formula for the Sthiti-khanda is $\frac{\sqrt{(R+r)^2-\beta^2}}{m_1-s_1}$ where β is the latitude of the Moon at the moment of conjunction. The value of MN at the moment of conjunction will not be equal to its value at any intermediate point because parallax in latitude differs from position to position of the Moon. In other words β is variable. The formulae given for the rectification of the Bhuja B or the Iṣtakāla I are

$$B' = \frac{B \times T'}{T} \text{ and } I' = T' - \frac{\sqrt{(R+r-g)^2-\beta^2}}{m_1-s_1} \times \frac{T}{T'}$$

T′ is the Sthiti-khanda rectified for the variability of β B′ is the Bhuja rectified for the same whereas T and B are the values of the Sthiti-khanda and Bhuja computed taking the effect of parallax in longitude above.

The effect of parallax in longitude is to prepone or postpone the moment of first contact as well as that of conjunction. The verse under commentary uses two terms Sphuta-Sthiti-khanda and Sphuteshuja-Sthiti-khanda. The former is the Sthiti-khanda rectified for parallax in longitude whereas the latter is that rectified for parallax in latitude ie. by adopting β' instead of β in the formula

$$\frac{\sqrt{R+r^2-\beta^2}}{m_1-s_1} \text{ where } \beta' = \beta \pm \text{ effect of parallax in latitude.}$$

Suppose on account of parallax in longitude the moment of first contact t_1 becomes $t_1+\delta t_1$, and let the moment of conjunction t_2 become $t_2+\delta t_2$. Then the Sthitikhanda unrectified for parallax in longitude will be (t_2-t_1) whereas that rectified for parallax will be $(t_2-t_1) + (\delta t_2-\delta t_1)$. This rectified Sthitikhanda is called Sphuta-Sthiti-khanda. Now the verse under commentary gives a procedure to rectify the Bhuja, and Iṣtakāla in the wake of β being effected by parallax in longitude. formula for Bhuja is $\frac{\sqrt{\overline{R+r-g}^2-\beta^2}}{m_1-s_1}$ wherein all quantities except β may be taken to be constant. Suppose β effected by parallax be comes β'. If $\beta' > \beta$, Bhuja will decrease; also the Sthiti-khanda whose formula is $\frac{\sqrt{(R+r)^2-\beta^2}}{m_1-s_1}$ decreases if $\beta' > \beta$. Hence if T' be the new value of T the Sthiti-khanda, $T' < T$. In other words when $\beta' > \beta$, $B' < B$ and $T' < T$. Also if $\beta' < \beta$, both B' and T' will be greater than B and T respectively. Hence as a rough measure B and T are taken to vary together positively or negatively and as such proportionally. Though both increase or both decrease together, strictly speaking the concept of proportionality is there; but roughly speaking they are taken to vary proportionally which means $B' = \frac{B \times T'}{T}$. The fact that proportionality is not there could be seen in two ways. $B = (T-I)(m_1-s_1)$ (1) with usual notation so that taking B and T to vary on account of the variation in β, $\delta B = \delta T\,(m_1-s_1)$, I not varying so that $B + \delta B = B' = (T+\delta T)\,(m_1-s_1) - I\,(m_1-s_1) = T'(m_1-s_1) - I\,(m_1-s_1) = (T'-I)(m_1-s_1)$ (2).

Dividing (1) by (2) $B/B' = \frac{T-I}{T'-I}$ which will be approximately equal to T/T' provided I is very small compared with T. Assuming so, B/B, could be taken to be equal to T/T, which means $B' = \frac{B \times T'}{T}$ as mentioned in the verse. Or again, considering the formulae for B and T and differentiating them with respect to β and getting B' and T', we shall have the following working.

$\frac{-g)^2-\beta^2}{-s_1}$ so that $2B\delta B =$

or $\delta B = \frac{-\beta\delta\beta}{B(m_1-s_1)}$; similarly $T^2 = \frac{(R+r)^2-\beta^2}{m_1-s_1}$ so that

$2T\,\delta T = \frac{-2\beta\,\delta\beta}{m}$ so that $\delta T = \frac{\beta\delta\beta}{T(m_1-s_1)}$

$\therefore\ B^1 = B+\delta B = B-\frac{\beta\delta\beta}{B(m_1-s_1)}$ and $T^1 =$

$T\cdot\ \ T(m_1-s_1)$

$$\therefore\ \frac{B^1}{T^1} = \frac{B^2-\beta\delta\beta}{B(m_1-s_1)} \times \frac{T(m_1-s_1)}{T^2}$$

$T-\frac{\beta\delta\beta}{T}$. Since $\frac{\beta\delta\beta}{B} \pm \frac{\beta\delta\beta}{T}$

$\frac{B^1}{T^1} \pm \frac{B}{T}$.

Regarding the finding of Iṣṭakāla when g the grāsa is given, we have the formula

$T-I = \frac{B}{m_1-s_1} = \frac{\sqrt{(R+r-g)^2-\beta^2}}{m_1-s}$ so that putting $T-I=t$

$\frac{\sqrt{R+r-g^2-\beta^2}}{m_1-s_1}$ $\therefore$ As β increases t decreases so that $T-t$ increases ie. I increases. Whereas as β increases T decreases. This means in a way that as T decreases. I increases so that I is taken to be inversely proportional to T ie. I' is taken to be $\frac{I\times T}{T'}$ as given.

Here also, it could be seen that the inverse proportionality is not strictly there; for β increasing both t and T decrease so that $T-t$ ie. I will increase only if the decrease in t is greater that in T. But t^2